A linear programming based model for strategic management of district heating systems

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SUMMARY

This paper is devoted to the development of a decision support system that could assist district heating authorities in strategic management of their asset. Such a tool is designed in order to answer a variety of questions asked by concerned stakeholders regarding the future of a district heating (DH) system about optimal plant and fuel choice, possible network extension, valorisation of heat surplus, etc.

The kernel of this system is a simulation model which determines the energy balance of a given DH network, over any type period. The time period is cut into different time steps, assuming steady state on each time step.

At each time step, a linear programming LP based approach is used in order to compute the optimal combination of heat power at the plants to meet the demand. The modelling problem is formulated on a graph and LP as the search for minimum energy production cost rooted at the sources.

INTRODUCTION

District heating (DH) is presently considered in Europe as a challenging technology, both as a great step toward sustainability of energy systems and as a powerful instrument for local energy policy managed by communities. The bases for establishing a DH supply is the possibility of obtaining higher efficiency and thereby lower heating costs when producing the heat in few large plants than when using small private boiler units. DH supply gives also the opportunity to utilise waste heat, that otherwise could not be used, especially from incineration plants. DH is also a good way to make efficient use of other local and renewable energy resources such as biomass, biogas, geothermy and to develop cogeneration. Moreover, environmental issues have lead to an even greater interest in DH supply, due to the possibility of cost-effective flue gas treatment in centralised plants, in order to limit local atmospheric pollution, or in the near future, CO_2 emissions.

The need for tools for decision making and strategic management is therefore obvious. The control and strategic management of a DH network is a complex process in which different factors like costs, availability, heat losses and environmental impacts must be taken into account. For this reason, mathematical aided models are being increasingly in use in order to identify the best decision factors in different operating situations and scenarios.

Examples of scenarios to be evaluated are : i) Refurbishment and extension of the DH system ; ii) Changing energy sources or plants ; iii) Variation of the heat demand ;
iv) Changes in control and operational management of the DH ; v) Defining new policies in heat pricing and connections. An important feature is also *ex-post* evaluation of network

operating, in particular when DH is operated by a private company, under authority of the community.

These models require appropriate mathematical description of the different subsystems which form the DH. Typical methods account for simulation by minimizing operational DH costs. A literature study has shown that several methods have been proposed for minimisation of the operational cost of DH systems, but only very few references include case studies where the suggested methods are general and could be applied to any network. Due to the great complexity of the problem, most of the references suggest methods which only regard some parts of the total problem, while other parts are not considered or treated separately. The proposed methods can, thus, be divided into two categories:

(1) Strategic and mid/long term models which includes two subgroups:

- Determination of the optimum load distribution between different heat producing units, in a well specified DH system: network topology, energy supply and heat plants are known. DH supply temperature is considered constant or predetermined and DH networks dynamics is disregarded. Approaches in this area include markedly the linear programming and mixed integer programming techniques as solving algorithms, at each time step [5] [6] [7]. This type of modelling cannot be actually considered as an optimisation approach (although an optimisation algorithm is performed), but rather a time-dependent simulation of a given DH system under specified conditions.

- Optimisation of the network structure by looking for the best construction and operational cost. These methods discuss namely the geometric optimization of the DH distribution system and the choice of the most cost-effective heat plants (to be build) and energy sources, accounting for investment and operation costs [9] [12]. Linear programming and mixed integer programming techniques are generally used as solving algorithms as well, but they incorporate a global cost optimisation over a long-time horizon.

(2) Fully dynamic optimization determining the optimum supply temperatures and system control, often related to operational studies. Some dynamic approaches, based on an aggregation of the DH network, are discussed by a number of authors [3] [4] [8], but a general method for solving the problem does not exist today.

The model proposed in this paper is aimed to emulate the DH running on a given period, either from the past, on the basis of historical data, or for scenarios of the future. The simulation model investigates the energy balance in each time step. The environmental and economic balances could be recognised. The model is a masterpiece of a strategic management tool that includes as well detailed data bases on the DH network e.g. outside temperature records, heat load, maximum heating power, heat losses, etc.

The model could assist decision makers in evaluating the impact of several strategic choices, by simulating the effect of different scenarios related to the introduction of new heating plants, new consumers, etc. Nevertheless, in a perspective of decision support, when the number of potential scenarios to be tested is high or a very large panel of actions is possible, the simulation model is not always sufficient. So it could be complemented and coupled with an optimization module in order to identify the optimal choice of plants, energy sources and

sizing of components. Anyway, the simulation model remains the kernel of the decision support tool.
For the real time control and operating, other tools based on thermo-hydraulic modelling or optimal dynamic control optimisation could be more reliable.

METHODS

The proposed model concerns a completely defined DH system, with given set of power plants and heating demand, and looks for determining the overall DH thermal balance and power flows in the DH network at each time step, under external conditions. As we will see, the model is based on a linear programming formulation. The decision variables are the heat flows in DH pipes. The constraints are of two kinds. First, thermal properties of heat plants and DH pipes will restrict the range of heat flows and second, distributed energy should satisfy consumer (substation) heating demand at any time period.

Analytical formulation

A suitable method to describe a district heating pipe system is to use concepts from network theory. We define a district heating graph G as a set of branches and nodes. A branch is assumed to be a pipe; a node could be a substation or a heating plant. Heat flow is assumed to be directed *a priori* from one node to another and the graph is oriented.

Notations

The following notations are used as subscripts

i, i'	for substations	$1 \le i \le n$
k	for heat plants	$1 \le k \le p$
j	for network branches (lines)	$1 \le j \le m$

A branch *j* connects two nodes, either a substation *i* to another substation *i'* or a heating plant *k* to a substation *i*. The corresponding information is provided through connectivity matrices, defined below. *Su(i)* is the set of outlet branches (successor) of node *i* (substation), *Pr(i)* is the set of inlet branches (predecessor) of node *i* ; *Su(k)* is the set of outlet branches of node *k (*heat plant). Heat plant nodes are assumed to be without inlet branches.

At a given time step, the following parameters are assumed to be input data

Δt	the time step length.
P_i^{apl}	heat demand in substation *i*.
P_k^{max}	maximum heat power of plant *k*.
c_k	cost of fuel used in plant *k*.
η_k	efficiency of plant *k*.
p_j^{max}	maximal heat power transferred through pipe *j*.
L_j	heat loss in branch *j*.

Unknown variables, to be determined for each time step, are the heat flows in the network

p_j	heat flow entering branch *j*.

Heat flow leaving a branch corresponds to (p_j - L_j), accounting for heat losses from pipes. Moreover, for each substation, two heat flows may be defined

P_i^- total heat flow leaving node *i*

$$P_i^- = \sum_{j \in Su(i)} p_j \, , \qquad (1)$$

P_i^+ total heat flow entering node ***i*** $$P_i^+ = \sum_{j \in \Pr(i)} p_j - \sum_{j \in \Pr(i)} L_j \,, \qquad (2)$$

For each substation, the heat demand from consumers must be met, so the thermal balance leads to

$$P_i^+ = P_i^- + P_i^{apl} \,, \qquad (3)$$

Principle

The proposed model to determine the state variables p_j is based on the minimization of the heat production cost of the whole district system over each time step of a work period, with respect to energy balance (both at substation and heat plants) and thermal pipe capacity constraints. While expressing the objective function and the constraints as function of heat flows, it can be seen that all equations are linear.

The problem is stated to be at each time step as follows:

$$\mathit{Minimize}\;\; f(p_j) = \sum_{k \in Sources} \sum_{j \in su(k)} \frac{c_k}{\eta_k} \times p_j \times \Delta t \,, \qquad (4)$$

With constraints

$$\sum_{j \in \Pr(i)} p_j - \sum_{j \in \Pr(i)} L_j = \sum_{j \in Su(i)} p_j + P_i^{apl} \,, \qquad (5)$$

$$\sum_{j \in Su(k)} p_j \le P_k^{\max} \,, \qquad (6)$$

$$0 \le p_j \le p_j^{\max} \,, \qquad (7)$$

Matrix formulation

Preliminary implementation of the model was done under MATLAB. A graph structure was designed under Matlab as a schematic network layout of the DH system topology and the LINPROG function was used for optimisation. Like many linear programming routines, LINPROG uses the standard simplex algorithm to solve LP problems and it requires data in an LP standard format which is the following:

To find out the vector *X* minimizing $f = {}^t c \,.\, X$ subject to

$$A \,.\, X = b$$
$$A' \,.\, X \le b'$$
$$u \le X \le v$$

Thus the inputs for this function (objective function, equalities and inequalities constraints) should be presented in a matrix format. Let's remind that we consider a DH system with *n* consumers (substations), *m* pipes and *p* heat plants.

The cost function *f* is written at each time step as a scalar product of two vectors *c* and *P*, where *c* contains the heat production costs (cost of fuel, divided by efficiency, + other possible proportional costs) and *P* is the unknown vector composed from the heat flows in the DH system pipes, p_j.

$$f = {}^t c \cdot P \,, \qquad (8)$$

Calculation of energy balance at each consumer (substation) is based on the first law of Kirchoff [11]. In order to express these equations in a matrix format we may define the connectivity matrix *A* of the DH representative graph as follows:

$$a(i,j)=\begin{cases}1, & \text{if the pipe } j \text{ is an incoming pipe for consumer } i.\\ -1, & \text{if the pipe } j \text{ is an outgoing pipe for consumer } i.\\ 0, & \text{if it's not connected.}\end{cases}, \qquad (9)$$

The connectivity matrix is an *n by m* matrix [1]: it has one column for each branch in the system and one row for each consumer. Each column can have two non zeros entries. The connectivity matrix expresses the inflows/outflows relationship in the district heating system.

Considering the energy balances equation at each substation (eqn. 3), the equality constraints of the DH model can be easily expressed as function of heat flows as:

$$A\,.\,P = b, \qquad (10)$$

Where b is the constant term, which contains the heating demand for each node, including heat losses from inlet branches.

$$b_i = P_i^{apl} + \sum_{j \in \Pr(i)} L_j \,, \qquad (11)$$

Inequality constraints express the inability of power plants to exceed a specific heating power value. We define as well a specific matrix denoted A' to present the inequality constraints in a linear format.

$$a'(k,j)=\begin{cases}1, & \text{if pipe } j \text{ is connected to source } k.\\ 0, & \text{otherwise.}\end{cases}, \qquad (12)$$

Hence, the inequality constraints could be expressed as function of the heat flows.

$$A'\,.\,X \le b'\,, \qquad (13)$$

Where b' contains the maximal heating power of each power plant.

The LP optimization routine LINPROG is called at each time step. The function inputs can be classified into two categories, variables and static parameters according to their values over each time step (see Fig. 1). Static inputs includes A and A' and they didn't depend on time since they are related to the network configuration. Variables parameters include the cost function f as well as second term parameters b and b'.

Typical simulation can be done with an hourly time step and a running period of one year, enabling to account for combined time variations of all input data (demand, energy price, heat plant characteristics). With additional assumptions on input data (constant demand, prices, availability, efficiencies), much longer time steps can be adopted, leading to faster simulation.

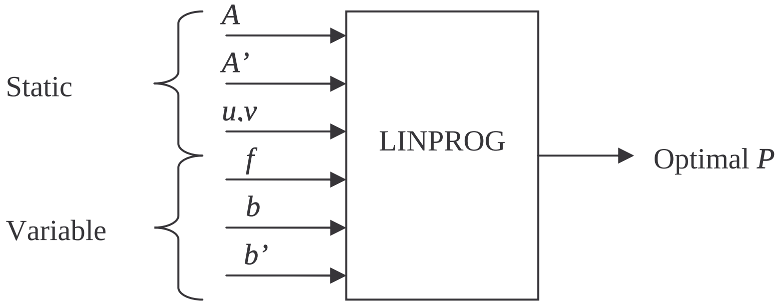

Figure 1. The optimisation routine, called at each time step

RESULTS

The aim of this section is to apply the proposed model in its hourly form to the district heating of Beaulieu Malakoff in Nantes. This district heating counts 20 km in length and 96 substations. The network provides approximately 115 GWh of energy to 16000 consumers. Incineration plants of municipal solid waste built in 1987 provides most of the heat for the network. The waste incinerator is modelled as an ordinary plant with variable maximum power and zero cost. The heat provided by an MSW incinerator is supposed to be totally free; the costs related to plant maintenance and waste transport are disregarded.
The maximum heating power of an MSW plant calculation is based on the lower heating value *LHV*(GJ/tons), the waste fuel rate *r* (tons/hour) and the boiler efficiency η thanks to this formula:

$$P_{\max} = LHV \times r \times \eta \,, \qquad (14)$$

The monthly values of LHV and waste fuel rate are used as inputs. Efficiencies are supposed to be constant and the availability of the plants is set to 100%.

The duration curve shows what proportion of heat production is attributed to each plant to satisfy the demand at any time. It may also give the start and stop moments in the heating plants.

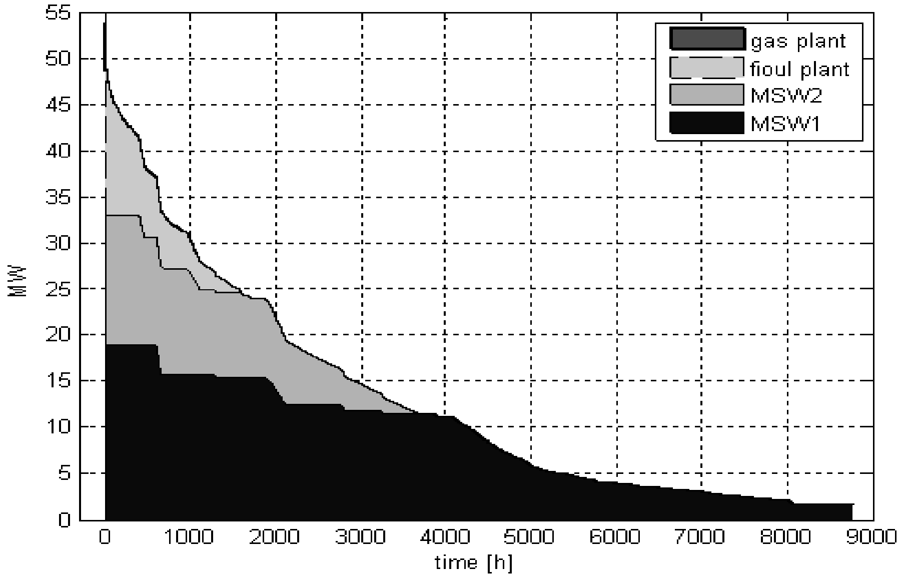

Figure 2. Load duration curve indicating the proportion of the heating plants

The duration curve as presented above shows in reality the production scheduling of the DH plants and their start and stop moments as function of heat load levels.

Table 1. Optimal heat production dispatching

Heat load range	Starting heating plant	Covering ratio
<12 MW	MSW1	67.8%
12-24 MW	MSW2	24%
24-49 MW	Fioul plant	7.9%
>49 MW	Gas plant	0.3%

Fossil basic plants are operational in peak loads; they cover the rest of the heat demand served basically by incinerators. The swap between the fossil heating plants depends obviously on the fuel cost. The model gives a priority to a boiler at a given moment if its fuel is the cheapest at that moment.

Table 2. Yearly balances (energy in GWh, costs in k€)

	Feed in primary side	Valorised heat	Heat surplus	tCO2 realised to the air	Running cost
Incinerators	235.9	105.3	95.2	60872	229.35
Fuel boiler	10.6	9	0	2774	
Gas boiler	0.02	0.015	0	4.19	

From a strategic point of view, the stakeholder could take decisions concerning heat production in the DH system in the future. For instance, assuming the same heat load for next years or using heat load forecasting techniques, the decision maker, by the aid of this model, will have an idea about the amount of energy in primary side required to cover the demand.

DISCUSSION

The mathematical model and algorithm presented in this work contributes as a first step toward the development of a fast, robust and accurate DH strategic planning tool. It can be considered as a general and flexible tool. In order to illustrate this flexibility, some examples are analysed below, considering the main features of district heating.

In term of **network characteristics**, the model is suitable whatever the topology of the network, hierarchical or looped network structure. Although the temperature of the network is not taken explicitly, DH systems with variable temperatures may be tackled. In most cases, flow and return temperatures depend on the total heat demand addressed to the network and can be computed at each time step, as an exogenous parameter (enabling to adjust input data varying with temperatures such as heat losses or heat plant characteristics). So, common network control strategies, either through fixed supply and return temperatures (and adjusted flow rate) or fixed flow rate (and adjusted supply temperature), can be considered by the model. Temporary problems taking place in the network, such as interruption or limitation of supply in a given branch, may be simulated by modifying the maximal heat power transferred through this pipe. Interconnection of a DH with other DH system(s) may also be considered easily. Pumping cost, which represents a non negligible fraction of operation costs (a few percents), can be modelled by giving a cost to heat transferred into branches where pumps are located.

Regarding the **energy demand** addressed to the network, the model may consider simplified demand profiles from consumer or very precise profiles from quite different types of consumers, depending only of the quality of available data and required accuracy. An interesting feature regarding demand is the possibility to simulate temporary and partial disconnection of some customers, using "virtual" heat plants associated to a cost of non-supply: interruption of demand is a service which can be provided by some customers to the network at a given contractual price.

Finally, the flexibility of the model concerns also **heat production plants**. A large variety of heat plant technologies are directly tackled, provided their maximal capacity and efficiency are known at each time step. This includes usual types of boilers, using fossil or renewable energy sources, whatever their cost. Waste heat, such as heat produced from domestic waste incineration plants, can be simulated, being considered as free energy source.

Availability of heat plants vs time (due to maintenance, e.g.) and time varying characteristics of the energy sources (price, lower heating value, maximum capacity) are considered through input parameters. For some heat plants, such as cogeneration, heat pumps or geothermal energy, their characteristics depend on network temperature. They may be simulated as well, as soon as this temperature is pre-determined at each time step.
The main limitation of the model is probably linked to the assumption of constant efficiency for each heat plant (considered as an input data for each time step, possibly varying with time). Efficiency depending on the load (with lower efficiency at partial load, e.g.) cannot be tackled by linear programming algorithm.

Thus, the model incorporates a lot of features, relevant for most of current district heating systems. It is being implemented as a master piece of a strategic management tool of district heating systems. In the same time, additional components are being considered for introduction in model equations, namely heat storage and complex cogeneration (including connection with electric grid).

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