Calculating the Heat Transfer of Wall Structures in Non-stationary Cases

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Summary

In order to get an accurate knowledge of the energy consumption of buildings and of comfort parameters inside of closed rooms depending on the time, we developed a mathematical model. Based on Fourier's differential equation, this mathematical model allows us to calculate the instantaneous heat loss, the amount of the heat flow stored in the wall between the two time values, and the plotting versus time of the temperature generated on the contact surface of a double-layer wall structure. In addition to the outside temperature, in the mathematical model we can take into account the radiation intensity reaching the external wall structures by way of sun radiation, as well as the plotting versus time of these meteorological characteristics. The boundary conditions of the developed equation system, which consists of 5 equations, can be determined on the basis of meteorological databases or our own measurements.

Introduction

For an accurate knowledge of the energy consumption of buildings and of comfort parameters inside of closed rooms depending on the time the development of mathematical models is necessary in the field of building structures and thus in the field of wall structures as well, which models enable the necessary calculations to be performed on a scientific basis but in a simple way and using relatively short CPU time. The results of these calculations are expected to provide the following numerical data both in winter and summer modes:

- The function describing the changing versus time of the heat loss in winter, and of the heat load in summer of the rooms, taking into consideration the thermal energy stored in the wall structures for some time.
- The values of the internal surface temperature of the wall structures, which allow us to evaluate the given room's thermal comfort if we know the other comfort parameters (air temperature, air velocity, humidity, clothes, activity level).

All of these calculated values should be determined with the knowledge of what we can regard the two most important meteorological variables, namely the outside temperature and the radiation intensity during any given day.

Methods

Calculating the heat transfer of wall structures in non-stationary cases is based on Fourier's differential equation. In addition to the necessary material characteristics (c: specific heat, ρ: density and λ: thermal conductance), this equation contains, as its last component, the effect of a heat source (E, in W/m^3), which allows us to also take into account the solar radiation reaching the wall surface.

$$c \cdot \rho \cdot \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda \cdot \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \cdot \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \cdot \frac{\partial t}{\partial z}\right) + E, \quad (1)$$

On the basis of this equation, and taking into consideration other factors as well, we can formulate an equation system consiting of 5 equations, two of which are differential equations that allow us to solve the equation system numerically if we choose the right steps.

The the equation system's unknown values, which have to be determined in relation to any given time period, are as follows:

- the value of the instantaneous heat loss: $\dot{Q}$,
- the heat flow stored in the wall between two time periods: $\Delta\dot{Q}$,
- the surface temperature of the internal wall structure: t_{iw},
- the temperature at the point of contact of the brick wall and of the heat insulation on its external surface: t_c.

In order to solve the equations, we need to know the meteorological characteristics, as well as the plotting versus time of the surface temperature of the external wall structures – out of these characteristics, the changing of the radiation intensity and of the air temperature for the given day was taken from an appropriate meteorological database [1], while the changing of the surface temperature of the external wall structures was determined by measurement.

Results

In order to define the mathematical model, we have used a double-layered wall structure that contains a brick structure capable of storing heat, and an outer heat insulation. Further, in relation to heating, we applied the following presumptions:

- By the wall's heat loss we mean the heat flow arriving at the internal wall structures, whose instatntaneous value, at the time of $\tau = 0$ is $\dot{Q}_0$.
- The heat flow stored in the wall in between the 1st and 2nd time period is: $\Delta\dot{Q}_1$.
- The heat flow on the external wall structures is the difference between the two quantities above, that is $(\dot{Q}_0 - \Delta\dot{Q}_1)$.

The mathematical model consists of the following equations:

$$\Delta\dot{Q}_1 = \frac{c_1 \cdot \rho_1 \cdot \Delta x_1}{6 \cdot \Delta\tau} \cdot \left[\left(t_{iw1} + t_{c1}\right) - \left(t_{iw0} + t_{c0}\right)\right] + \frac{c_2 \cdot \rho_2 \cdot \Delta x_2}{6 \cdot \Delta\tau} \cdot \left[\left(t_{c1} + t_{ew1}\right) - \left(t_{c0} + t_{ew0}\right)\right] + \\ + \frac{\lambda_1}{3} \cdot \frac{\left(t_{iw0} - t_{c0}\right)}{\Delta x_1} - \frac{\lambda_2}{3} \cdot \frac{\left(t_{c0} - t_{ew0}\right)}{\Delta x_2} + \frac{I_r \cdot a}{3} + \frac{\alpha_i}{3} \cdot \left(t_{i0} - t_{iw0}\right) - \frac{\alpha_e}{3} \cdot \left(t_{ew0} - t_{e0}\right) \quad (2)$$

$$\dot{Q}_0 = \alpha_i \cdot \left(t_{i0} - t_{iw0}\right), \quad (3)$$

$$\dot{Q}_0 = \frac{c_1 \cdot \rho_1 \cdot \Delta x_1}{6 \cdot \Delta \tau} \cdot \left[\left(t_{iw1} + t_{c1} \right) - \left(t_{iw0} + t_{c0} \right) \right] + \frac{c_2 \cdot \rho_2 \cdot \Delta x_2}{6 \cdot \Delta \tau} \cdot \left[\left(t_{c1} + t_{ew1} \right) - \left(t_{c0} + t_{ew0} \right) \right] +$$
$$+ \frac{\lambda_1}{3} \cdot \frac{\left(t_{iw0} - t_{c0} \right)}{\Delta x_1} - \frac{\lambda_2}{3} \cdot \frac{\left(t_{c0} - t_{ew0} \right)}{\Delta x_2} + \frac{I_r \cdot a}{3} + \frac{\alpha_i}{3} \cdot \left(t_{i0} - t_{iw0} \right) + \frac{2 \cdot \alpha_e}{3} \cdot \left(t_{ew0} - t_{e0} \right) \quad (4)$$

$$t_{c0} = \frac{\lambda_1 \cdot \Delta x_2 \cdot t_{iw0} + \lambda_2 \cdot \Delta x_1 \cdot t_{ew0}}{\lambda_2 \cdot \Delta x_1 + \lambda_1 \cdot \Delta x_2}, \quad (5)$$

$$\Delta \dot{Q}_{j+1} = \dot{Q}_{j+1} - \dot{Q}_j, \quad (6)$$

The indexes used in the equations indicate the following:

1 and 2	the values of the brick wall and the heat insulation
iw and ew	internal and external wall structures
c	contact point between the brick wall and the heat insulation
i and e	internal and external
r	radiation
0, 1, j	time periods and running index

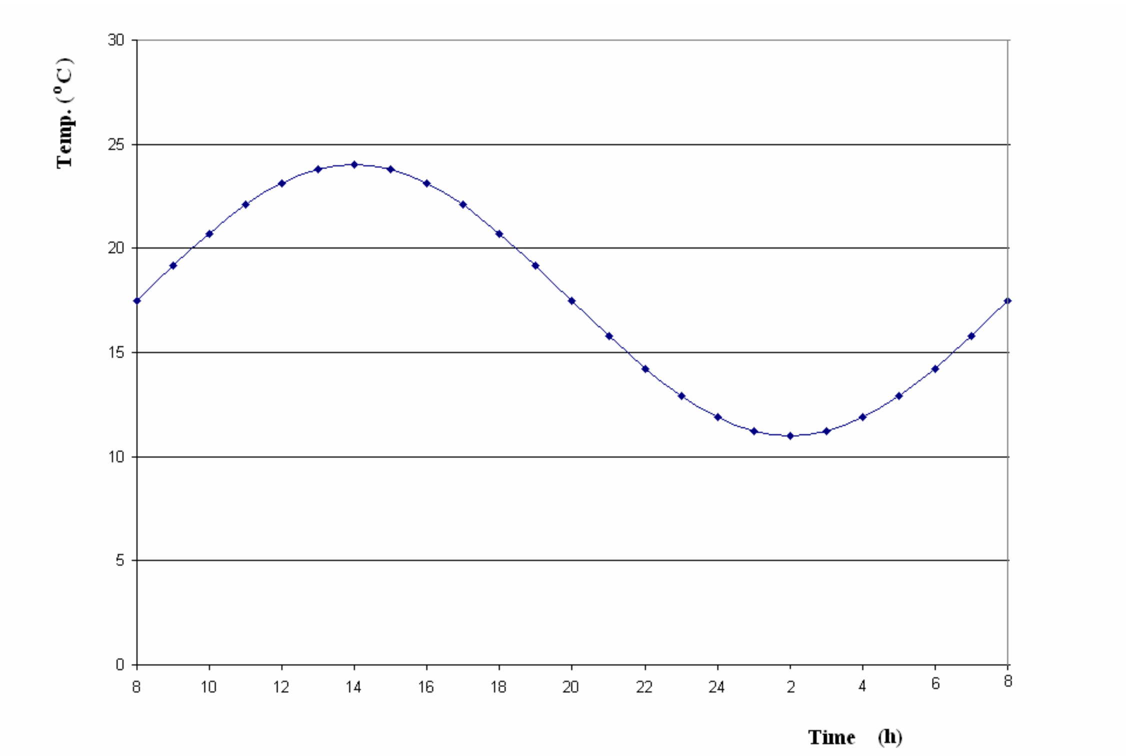

Figure 1. Daily outside temperature fluctuation in Hungary during the month of September

Figure 1 shows the external temperature characteristics of the month of September in case of clear weather. While analyzing the meteorological data, we found that, given Hungary's climate conditions, the average daily temperature measured at 8 am in the morning is the same in the case of clear or cloudy weather. In view of this fact, we determined the following equation for the plotting versus time of the temperature during a given day:

$$t = t_m + A \cdot \sin \frac{2 \cdot \pi}{24} (\tau - 8), \quad (7)$$

Where t_m is the average daily temperature, while A is the amplitude of the daily temperature fluctuation.

Figures 2 and 3 show the daily temperatures measured on the internal and external surfaces of the wall structures facing East. (The wall structure was made of 30 cm wide perforated bricks, plastered on both sides.) The daily fluctuation of the radiation intensity is described on the basis of curves that connect the time periods with the same or similar radiation intensity [1], Figures 4.

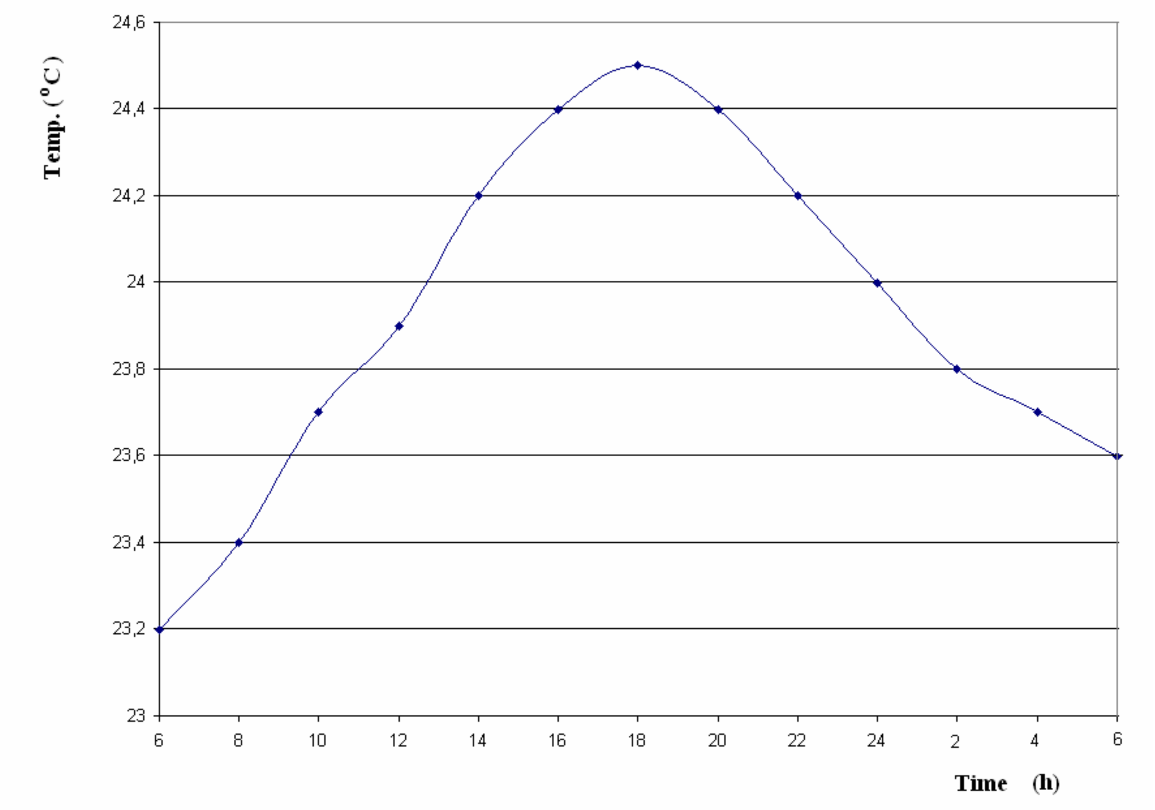

Figure 2. Daily surface temperature fluctuation on the internal wall surface of the wall structure facing East, measured in September 2006

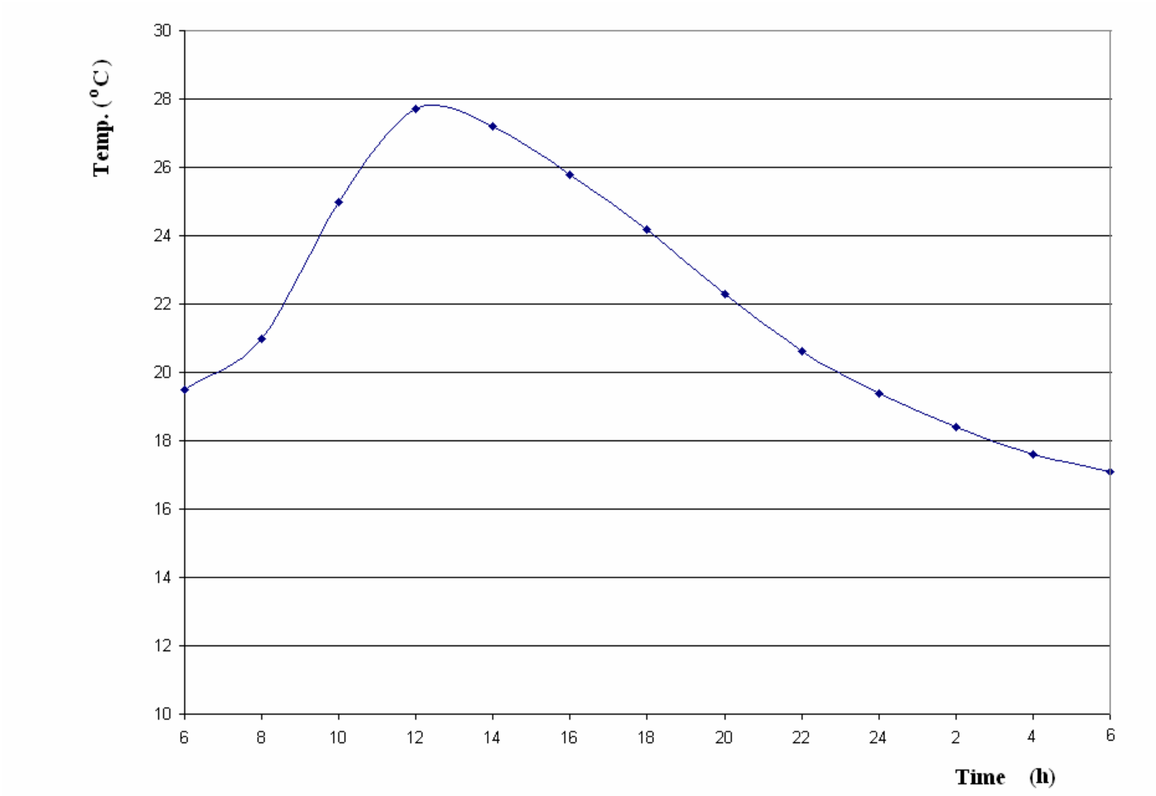

Figure 3. Daily surface temperature fluctuation on the external wall surface of the wall structure facing East, measured in September 2006

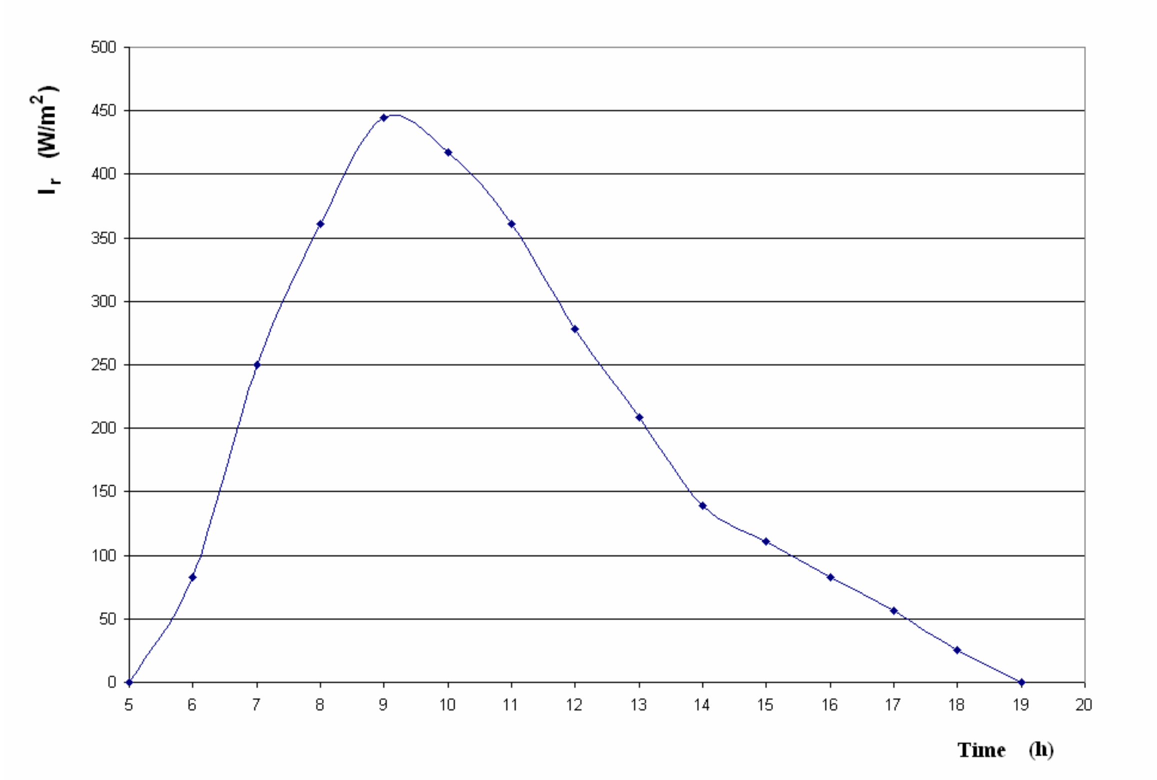

Figure 4. Daily fluctuation of radiation intensity in Hungary, on an average day in September

Discussion

To sum up the research work, we can draw the following conclusions:

- The equation system defined in (2) – (6) can be solved if we know the boundary conditions as defined on the basis of measurements and meteorological databases. The equation is best solved by applying a computer algebra system. When recording the $\Delta\tau$ = 1 hour interval, we need to write down a total of 122 equations for one day, which allows us to define the unknown factors.
- From the measuring results of the internal and external wall surface temperatures the damping effect of the brick wall becomes quite obvious: while on the external wall structures we can detect approximately 11 °C daily fluctuation, this value on the internal wall structures is only 1,2 °C.
- As shown in Figures 2 and 3, after a cooler night (in comparison to the preceding nights), the value of t_{iw} will not drop to the value of the previous period's, but will increase to a small extent (approximately 0,3 °C). This temperature difference will be covered by the thermal energy that had been previously stored in the wall structure.

References

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