Modeling the Heat Gain of a Window With an Interior Shade, How Much Energy Really Gets In?

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ABSTRACT

Not long ago the ASHRAE Technical Committee on Load Calculation, TC 4.1, had a “bake off” of sorts between different peak air-conditioning load calculation schemes and programs. One of the outcomes of this exercise was the realization that practitioners and software developers make largely different assumptions about how solar energy absorbed by window glass and by window shades contributes to the room solar heat gain. For unshaded glass windows there is general agreement [ASHRAE Handbook of Fundamentals, 2005]. For a shade however, there were two extremes in the models—one assumes that the shade rejects most of the solar energy that it does not transmit, logical if the shade is highly reflective and the glass highly transmissive, the other assumes that all radiation absorbed by the shade is immediately convected into the room.

Suffice it to say that we are not the first to derive and present the fundamental equations of this heat transfer problem. What we have done is to avoid any simplifying assumptions in formulating the problem while allowing that some physical constants, convection coefficients in particular, are not well known and need to be parameterized. We whet the readers’ appetite by revealing that for a glass/shade system where the glass was 22% transmissive and the shade 52% transmissive, the total heat gain to the room from this window assembly was nearly half of the incident radiation. Of course “It all depends!”

INTRODUCTION

Not long ago the ASHRAE Technical Committee on Load Calculation, TC 4.1, had a “bake off” of sorts between different peak air-conditioning load calculation schemes and programs. One of the outcomes of this exercise was the realization that practitioners and software developers make largely different assumptions about how solar energy absorbed by window glass and by window shades contributes to the room solar heat gain. For unshaded glass windows there is general agreement [ASHRAE Handbook of Fundamentals, 2005]. For a shade however, there were two extremes in the models—one assumes that the shade rejects most of the solar energy that it does not transmit, logical if the shade is highly reflective and the glass highly transmissive, the other assumes that all radiation absorbed by the shade is immediately convected into the room.

Suffice it to say that we are not the first to derive and present the fundamental equations of this heat transfer problem (see [McCluney and Mills, 1993, and Duffie and Beckman, 1991]). What we have done is to avoid any simplifying assumptions in formulating the problem while allowing that some physical constants, convection coefficients in particular, are not well known and need to be parameterized. We whet the readers’ appetite by revealing that for a glass/shade system where the glass was 22% transmissive and the shade 52% transmissive, the total heat gain to the
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**OBJECTIVE**

The objective of this study was to model the true solar heat gain of glass and shade. Older, rough calculations have assumed that a 50% opaque shade will reduce the solar heat gain through the glazing by something just less than 50%. One of the less well understood parts of these estimates is the energy convected from the glass and shade into the room.

The model developed under this study provides an accurate and complete calculation scheme to support the performance of a shade having various transmissivities and absorptivities. The absorptivities are important for the calculation of solar interaction between the inside surface of the glass and shade and to determine the temperature of the shade facing the occupants in a room. More importantly, inside and outside surface convection coefficients can be varied to establish limits on how they impact solar heat gain.

**BACKGROUND**

We know that if a porous shade is suspended between the inside surface of the glass and the adjoining room, the solar radiation will be reduced. However a detailed model is required to address several issues. These include: How much energy is reflected between the outer surface of the shade and the inner surface of the glass and what happens to this energy? How much energy is re-radiated back into the room through the shade? How much energy is convected from the glass and shade into the room.

**THE MATHEMATICAL CALCULATIONS**

The model developed permits the parametric evaluation of the window/shading characteristics including: The effect of different transmissivities of the shade. The surface temperature of the glass with the shade in place. The surface temperature of the shade in the cavity. Convection coefficients, both inside and outside.

Note that in the model the effects of the sash and frame have been excluded. However, ASHRAE Standards 90.1 and 90.2 and national Energy Codes require that fenestration ratings be determined in accordance with NFRC (National Fenestration Rating Council) procedures. These procedures (NFRC 200 for solar heat gain coefficient) explicitly rate the entire fenestration product including the frame, sash, and glass. See the NFRC website at www.nfrc.org for further information.

*Modeling Incident solar radiation (this section is presented for the sake of completeness)*

The solar constant, $G_{sc}$, is the energy from the sun, per unit time, received on a unit area of surface perpendicular to the direction of propagation of the radiation, at mean earth-sun distance, outside of the atmosphere. Measurements of the constant were made with high altitude aircraft flights, balloons, and spacecraft [Thekaedara et.al. 1971 and 1976]. Later, spacecraft and rocket flight data were reported [see Hickey et.al, Wilson et.al., and Duncan et al.] and led to the adoption of the value of 1367 W/m² for the solar constant by the World Radiation Center. We will use this value to calculate the beam, diffuse and reflected radiation on the exterior of a window.
The distance from the earth to the sun varies with time of year. Hence the extraterrestrial radiation, \( G_{on} \), measured on the plane normal to the radiation on the \( n \)th day of the year is given by:

\[
G_{on} = G_{sc}(1 + 0.033\cos\frac{360n}{365}) \tag{1}
\]

Clear sky radiation on the earth’s surface can be estimated by calculating the atmospheric transmittance for beam radiation, \( \tau_{batmos} \), from a method presented by Hottel [Hottel, 1976],

\[
\tau_{batmos} = a_o + a_1 \exp(-k / \cos \theta_z) \tag{2}
\]

\( \theta_z \) = solar zenith angle, \( a_o=0.3887 \), \( a_1=0.5496 \), \( k=0.3176 \), (for mid-latitude winter)

Liu and Jordan, 1960 formulated an empirical relationship for diffuse radiation for clear days:

\[
\tau_{datmos} = \frac{G_d}{G_o} = 0.271 - 0.294 \tau_b \tag{3}
\]

where \( G_o=G_{on}(\cos(\theta_z)) \).

As an example, for a clear day on December 21\textsuperscript{st} in Los Angeles\textsuperscript{1}, the above formulations yield:

\( \tau_{batmos}=0.6938 \) –atmospheric transmittance beam radiation, \( \tau_{datmos}=0.0670 \) –atmospheric transmittance diffuse radiation, \( G_{on}=1411 \text{ W/m}^2 \) –extraterrestrial normal beam radiation, \( G_{onb}=979.3 \text{ W/m}^2 \) --normal beam radiation at the earth’s surface, \( G_o=761.5 \text{ W/m}^2 \) --extraterrestrial beam radiation on a horizontal plane, \( G_{od}=94.59 \text{ W/m}^2 \) --diffuse radiation at the earth’s surface.

**Beam and Diffuse Radiation Transmitted and Absorbed**

Figure 1 shows a schematic of the glass and porous shade system. The arrows are a symbolic representation of the short wave energy, accounting for radiation, transmitted, reflected, and absorbed. The diagram is for beam radiation but a similar accounting applies to diffuse radiation.

“Abs” denotes the fraction of absorbed short wave energy for the glass or shade on each successive reflection. The portion of the incident beam short wave energy that is ultimately absorbed by the shade, \( \text{Abs}_{bss} \), is:

\[
\text{Abs}_{bss} = \tau_{gb} \alpha_{ss} \sum_{n=0}^{\infty} \left[ \rho_{ss}\rho_{gd} \right]^n = \frac{\tau_{gb} \alpha_{ss}}{1-\rho_{ss}\rho_{gd}} \tag{4}
\]

\( \tau_{gb} \)–short wave transmittance of the glass (a function of incidence angle, \( \theta \)), \( \alpha_{ss} \)–short wave absorptance of the shade, \( \rho_{gd} \)–diffuse reflectance of the glass\textsuperscript{2}

\textsuperscript{1} We use this example location and date throughout the paper a representing a hot winter climate when south sun will be at a maximum. Also, accounting for cloudy weather is possible using the model provided but since the paper is motivated by and concerned with peak load calculation we have not considered cloudy days in our example.

\textsuperscript{2}Brandemuehl and Beckman have shown that the effective incidence angle for isotropic diffuse radiation is approximately 60 degrees. [Brandemuehl and Beckman, 1980]. integration performed using manufacturer’s data shown in the next section produced results that agree to within 1% of the Brandemuehl/Beckman results.
The incident radiation on the inside of the glass is diffuse radiation reflected from the shade. Optical properties for diffuse radiation for the glass can be determined for isotropic radiation by integration over all angles of incidence. This was done using glass properties from a manufacturer and shown in the next section of this report.

Similarly, the short wave beam radiation fraction absorbed by the glass, $\text{Abs}_{bg}$, is:

$$\text{Abs}_{bg} = \alpha_{gb} + \tau_{gb} \alpha_{gd} \sum_{n=0}^{\infty} \left[ \rho_{ss}^n + \rho_{gd} \right] = \alpha_{gb} + \tau_{gb} \frac{\alpha_{gd} \rho_{ss}}{1 - \rho_{ss} \rho_{gd}}$$  \hspace{1cm} \text{Eq}[5]$$

where variables are as defined before and $\alpha_{gb}$ = beam absorptance of the glass (a function of incidence angle, $\theta$), $\alpha_{gd}$ = diffuse absorptance of glass. Equations 4 and 5 also apply for diffuse radiation $\text{Abs}_{dss}$ and $\text{Abs}_{dg}$, but $\tau_{gb}$ is replaced with $\tau_{gd}$ and $\alpha_{gb}$ is replaced with $\alpha_{gd}$ (see footnote 7).

Figure 1. Short wave energy flows.

We can now do a short wave energy balance:

$$\dot{Q}_{gs} = \left[ G_{enb} \cos \theta \text{Abs}_{bg} + G_{cd} \text{Abs}_{dg} \right]$$ \hspace{1cm} \text{Eq}[6]$$

$$\dot{Q}_{ss} = \left[ G_{enb} \cos \theta \text{Abs}_{dss} + G_{cd} \text{Abs}_{dss} \right]$$ \hspace{1cm} \text{Eq}[7]$$

\footnote{3 We assume a uniform diffuse short wave field. This is accurate except for the lower floors where the diffuse radiation is a mix of diffuse sky radiation and ground reflected radiation. We have ignored ground reflected radiation in this analysis but its inclusion would be straightforward.}
P = porosity or open fraction of the shade (this is the transmittance of the shade), \( \dot{Q}_\text{gs} \) and \( \dot{Q}_\alpha \) are the short wave radiation absorbed by the glass and shade respectively. Some of the radiation reflected from the shade is reflected again by the glass and then transmitted through the shade to the room. Referring to Figure 3 the quantity is:

\[
\dot{Q}_\text{refin} = G_{\text{cnb}} \cos\theta \left[ \frac{P \cdot \tau_{\text{gs}}}{(1 - \rho_{\text{ss}} \cdot \rho_{\text{sd}})} - P \cdot \tau_{\text{gs}} \right] + G_{\text{cd}} \left[ \frac{P \cdot \tau_{\text{gs}}}{(1 - \rho_{\text{ss}} \cdot \rho_{\text{sd}})} - P \cdot \tau_{\text{gs}} \right]
\]

\text{Eq}[8]

**Glass Properties**

Data has been obtained for a heat absorbing glass. This product represents an application where total heat gain might be significantly higher than intuition may suggest because its transmissivity is so low. We will begin our analysis with this product but will move on to others. Figure 2 shows the transmittance, absorptance, and reflectance for the glass as a function of incident angle. In general, these properties can be characterized as functions of the cosine of the incidence angle, \( \Theta \). An incidence angle modifier can be defined as the ratio of the property at angle of incidence \( \Theta \) to the property at normal incidence.\(^4\) For example:

\[
K_{\text{transmittance}} = \frac{\tau_{\theta}}{\tau_{n}}
\]

\text{Eq}[9]

Plots of incidence angle modifiers for transmittance, absorptance, and reflectance versus \( \frac{1}{\cos(\theta)} - 1 \) are shown in Figure 3. Curve fits are shown for transmittance and reflectance – the fits are generally excellent. The equations for the trend lines can be used to calculate transmittance and reflectance for any angle \( \Theta \) less than 80 degrees and, since \( \tau + \alpha + \rho = 1 \), the absorptance can be calculated. For a winter design day at noon in Los Angeles, \( \Theta \) is 22.6 degrees.

\(^4\) The incidence angle modifier (IAM) has its roots in the experimental characterization of solar collectors. The definition used here is a slight modification of the IAM used with solar collectors.
Solar Transmittance, Absorptance, Reflectance vs Theta

Figure 2. Optical properties of heat absorbing glass.

Figure 3. Incidence angle modifiers versus \((1/\cos(\theta)) - 1\)
**Long Wave Radiation Exchange**

The long wave radiation exchange between the glass and the shade can be characterized by treating the problem as one of gray body radiation exchange with essentially the same geometric interpretation as for short wave radiation. The interior is assumed to behave like a cavity and hence a black body. The long wave radiation heat exchange between the glass and shade can then be written as:

\[
\dot{Q}_{gsl} = \frac{\varepsilon_{ssl}(1-P)\varepsilon_{gl}\sigma(T_g^4 - T_s^4)}{1 - (1 - \varepsilon_{gl})(1 - P)(1 - \varepsilon_{ssl})} \text{ W/m}^2 \text{ into the mesh} \quad \text{Eq}[10]
\]

where \(\sigma\) = Boltzman constant = 5.670E-08 W/m\(^2\)-K\(^4\) and \(T_g\) and \(T_s\) are the glass and shade temperatures in degrees Kelvin. Note that the conductivity of the glass and shade are sufficiently high that their temperatures can be assumed to be locally uniform. Similarly,

\[
\dot{Q}_{s lg} = -\dot{Q}_{g sl} \text{ W/m}^2 \text{ into the glass} \quad \text{Eq}[11]
\]

The long wave radiation from the interior to the glass is:

\[
\dot{Q}_{igl} = \frac{P\varepsilon(T_i^4 - T_g^4)}{1 - (1 - \varepsilon_{gl})(1 - P)(1 - \varepsilon_{gl})} \text{ W/m}^2 \text{ into the glass} \quad \text{Eq}[12]
\]

The glass also exchanges long wave radiation with the outdoors. Assuming that all the surroundings are at the outdoor temperature, the long wave radiation received by the glass from the outdoor environment is:

\[
\dot{Q}_{ogl} = \varepsilon_{gl}\sigma(T_o^4 - T_g^4) \quad \text{Eq}[13]
\]

Similarly, the radiation from the interior to the shade can be written as:

\[
\dot{Q}_{isl} = \frac{P(1 - \varepsilon_{gl})(1 - P)\varepsilon_{ssl}\sigma(T_i^4 - T_g^4)}{1 - (1 - \varepsilon_{gl})(1 - P)(1 - \varepsilon_{ssl})} + (1 - P)\varepsilon_{ssl}\sigma(T_i^4 - T_s^4) \quad \text{Eq}[14]
\]

Hence the total incoming long wave radiation for the glass is:

\[
\dot{Q}_{gl} = \dot{Q}_{ogl} + \dot{Q}_{s lg} + \dot{Q}_{igl} \quad \text{Eq}[15]
\]

and for the shade:

\[
\dot{Q}_{sl} = \dot{Q}_{gsl} + \dot{Q}_{isl} \quad \text{Eq}[16]
\]

**Convection**

Heat is transferred from or to the glass and shade by convection to the interior air and outdoor air. For the glass, heat flow in by convection is:

\[
\dot{Q}_{gc} = h_i(T_i - T_g) + h_o(T_o - T_g) \quad \text{Eq}[17]
\]
where \( h_i \) = interior convective heat transfer coefficient W/m²-K (5 was used initially), \( h_o \) = exterior convective heat transfer coefficient W/m²-K (15 was used initially). For the shade:

\[
Q_{sc} = h_i 2(1 - P)(T_i - T_g)
\]

**Total Heat Balance**

We now note that the heat flows for long and short wave radiation and for convection have been defined as being into the glass and shade. Since the sum of the energy into the glass and into the shade has to be zero in steady state, some flows will have a negative sign indicating that the heat flow is out of the material. The overall grand total heat balance for the glass is:

\[
\dot{Q}_g = \dot{Q}_{gs} + \dot{Q}_{gl} + \dot{Q}_{gc} = 0
\]

For the shade:

\[
\dot{Q}_s = \dot{Q}_{ss} + \dot{Q}_{sl} + \dot{Q}_{sc} = 0
\]

(As a reminder, all flows and heat balances are based on a unit area of the exterior façade, i.e. one square meter of glass adjacent to one square meter of shade)

The total radiant heat flow transmitted and reradiated into the room is:

\[
\dot{Q}_{gainrad} = -\dot{Q}_{isl} - \dot{Q}_{lgl} + P(G_{cnb} \cos(0))\tau_{gs} + G_{cd}\tau_{gld} + \dot{Q}_{refin}
\]

The total heat flow into the room caused by radiation and convection from/to the glass and shade is:

\[
\dot{Q}_{gan\text{t ot}} = \dot{Q}_{gainrad} - h_i (T_i - T_g) - \dot{Q}_{sc}
\]

**Engineering Equation Solver**

The model, as defined by the above equations, was implemented in Engineering Equation Solver (EES), a tool for solving simultaneous equations, especially those involving solar energy relations and thermodynamic properties [Engineering Equation Solver, 2005].

**GENERAL RESULTS**

The results are from a simulation of a south facing room on a hot winter day in Los Angeles (outside temperature 35 °C, 95 °F). This illustrates how the model can be used. The glass in this example was assumed to be oriented vertically and facing due south. However, any tilt or azimuth angle can be explored using the EES model. Energy flows for the glass/shade ensemble were enumerated for this day and are shown in Figure 4 below.

The figure shows that about 45% of the incident radiation becomes a heat gain to the room. This occurs even though the transmittance of the glass is about 0.22 and the transmittance of the shade is 0.53. The long wave radiation and energy convected into the room from the glass and shade contribute significantly to the heat gain (in addition to what is transmitted directly). Multiple reflections between the shade and glass, sometimes not considered in simple models, also impact heat gain, both directly and as they affect the surface temperatures of the glass and shade.

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5 Convection coefficients from the ASHRAE Handbook of Fundamentals were used as starting points but the ASHRAE values are higher than those used because they include equivalent radiant transfer, which is handled explicitly in this model. See ASHRAE Handbook of Fundamentals. Even taking this into account both these coefficients may be high and we will return to this issue later.
This study also evaluated the affects of shade “porosity,” the fraction of open area of the shade or its transmissivity. Their impact was evaluated using parametric tables. Figure 5 shows the variation of glass and shade temperature with variation of porosity. For the shade, more energy is absorbed with decreasing porosity but there is a simultaneous increase in energy convected off of the shade due to the increased surface area of the shade material. The glass, on the other hand, receives more reflected radiation from the shade at low porosity causing a very slight increase in glass temperature. However, the glass temperature is most strongly influenced by its high absorptance in the solar spectrum.

Figure 6 shows the total heat gain to the room and the radiant heat gain versus shade porosity. The relationship is very nearly a straight line ranging from about 415 to 520 W/m² (130 Btu/hr-ft² to 166 Btu/hr-ft²). At the lower end, heat transfer to the room is dominated by convection from the glass and shade and by long wave radiation transfer. At the upper end, direct solar gain plays a more important role. Long wave radiation from the glass to the room also increases as the glass “sees” more of the room and less of the shade. The glass absorbs a large fraction of the incoming radiation and hence is hot, causing long wave radiation to go into the room and promoting convective transfer from the glass to the room.

Some observations are in order. First it is important to say that the heat absorbing glass used in our model represents a rather extreme case – it really is a solar collector. That said, we note from figure 4 that 45% of the heat gain to the room is convected from the glass and shade. Furthermore, we used table 22 of chapter 31 of the 2005 ASHRAE Handbook of Fundamentals to find the solar heat gain coefficient and the interior attenuation coefficient and to calculate the fraction of incident radiation that would become heat gain. For a shade transmissivity of 0.2 the result was 0.29. The model described above predicts 0.43.
Figure 4. Energy flows occurring in the glass/shade system.
Figure 5. Glass and shade temperature versus shade porosity (transmittance).

Figure 6. Total heat gain through façade versus shade porosity (transmittance).

**Convection coefficients**

We next explore the impact of convection coefficients on total solar heat gain. As a reminder, the properties of the glass and shade are:

<table>
<thead>
<tr>
<th></th>
<th>Glass</th>
<th>Shade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmissivity</td>
<td>0.23</td>
<td>0.53</td>
</tr>
<tr>
<td>Reflectivity</td>
<td>0.11</td>
<td>0.22</td>
</tr>
<tr>
<td>Absorptivity</td>
<td>0.66</td>
<td>0.25</td>
</tr>
</tbody>
</table>
We deal first with the outside convection coefficient. Yazdanian and Klems [1994] measured the convection coefficient on their test facility for a first floor window and determined that it varied from 2-20 W/m²·K over 0-10 m/s (0-45 miles per hour). Keeping 5 W/m²·K for the inside convection coefficient, the variation in heat gain due to variation in outside convection coefficient can be calculated and is shown in figure 7.

![Figure 6. Heat gain versus outside convection coefficient.](image)

How does figure 7 illuminate us. Firstly, we are reminded that the model we have developed is for total heat gain through a glass/shade system. In the dark, at night, there would be heat gain from a hot outdoors to a cool room and a low convection coefficient would reduce the heat gain. However, when the glass is hotter than the surroundings, wind (and a high outside convection coefficient) helps reduce the heat gain to the room even when it is hot outdoors and cool indoors.

In this example with heat absorbing glass, the worst case scenario is not when the exterior film coefficient is high (i.e. a high speed wind) but when the air outside the glass is still. Methods that try to separate conduction/convective heat gain due to temperature difference through a window from solar heat gain seemed doomed or at least impractical. Ditto for schemes that try to combine convection and radiation at the inside or outside surface.

Figure 7 shows that heat gain to a room caused by the sun can vary by 50% depending on how much energy absorbed by the glass is swept to the outdoors. How about convection off the inside of the glass and shade? Fisher and Pederson [1995] and Spitler in referenced work show that the inside convection coefficient off interior walls ranges between 2 and 5 W/m²·K. If, for the sake of consistency, we assume that the outside convection coefficient is 15 W/m²·K, what happens when we vary the inside coefficient? Figure 8 below shows the result. We varied the inside coefficient from 2 to 10 W/m²·K because the glass and shade could be hotter than other room surfaces, promoting convection. The variation is 45%.

We now compare heat absorbing glass with 1/8 inch clear glass in the figures below. Figure 9 shows the total heat gain to the room for each glass/shade system. The clear glass is more
transmissive and the glass/shade system admits more heat to the room. It is not quite as simple as it may seem however as figure 10 reveals the temperature of the glass and shade for the two glass/shade systems. For the heat absorbing glass/shade system, the glass is hotter than the shade. For the clear glass/shade system the opposite is true, the shade is hotter than the glass. From the point of view of convective heat gain, most comes from the glass in the first case most comes from the shade in the second case.

Figure 7. Window heat gain for heat absorbing and clear glass

Figure 8. Window heat gain versus inside convection coefficient.
Figure 11. Solar heat gain versus glass transmissivity, reflectivity of 0.11, shade transmissivity of 0.53.

Finally, figure 11 shows the impact of the variation of transmissivity of the glass from 0.23 to 0.80, keeping the reflectivity of the glass at 0.11 and the shade transmissivity at 0.53. The relationship is linear.

**CONCLUSIONS**

Modeling results lead to the following conclusions:

For the baseline glass/shade system studied here, the temperature of the glass and shade vary with shade transmissivity, the glass varying from 51.4 to 55.5 C (124.6 to 131.8 F) and the shade changing from 34.4 to 37.6 C (95.8-99 F).

Again for the baseline glass/shade system, the heat gain through the south façade varies almost linearly with shade transmissivity, ranging from about 416 W/m² (132 Btu/hr-ft²) to about 521 W/m² (165 Btu/hr-ft²) as the transmissivity varies from 0.1 to 0.85.
The transmissivity of glass and a shading device do not, by themselves, define the solar heat gain into the room. The reflectance of the glass is equally important. Long wave radiant exchange and convection from the glass and shade to the room are significant components of heat gain.

For the hot winter condition studied, the solar heat gain is predicted to be 480 W/m² (152 Btu/hr-ft²). The contribution of this heat gain to the cooling load should be calculated using the load calculation methods described in the ASHRAE Handbook of Fundamentals, 2001.

**FURTHER WORK**

ASHRAE is currently sponsoring work to develop more complete models and verify them experimentally.

**REFERENCES**