

Derivation and analysis of the outdoor Wet Bulb Globe Temperature index (WBGT) with a human thermal engineering approach — Part 2. Properties of the WBGT formula for outdoor conditions with solar radiation

Kouhei Kuwabara¹, Tohru Mochida¹ and Tomonori Sakoi²

¹Hokkaido University, Japan

²National Institute of Advanced Industrial Science and Technology, Japan

Corresponding email: kuwa@eng.hokudai.ac.jp

SUMMARY

The authors present a theoretical derivation of the WBGT formula for outdoor conditions that was originally developed from the results of experiments on human subjects, based on a heat balance equation between the human body and its outdoor environment. The three coefficients of wet bulb temperature T_w , globe temperature T_g , and air temperature T_a were expressed in almost the same way as in the indoor WBGT formula, but they contain a new element characterizing solar radiation. In addition, we calculated the coefficients in the theoretically derived formula, changing the amounts of metabolic activity, clothing worn, wind velocity, and solar radiation. We obtained the new formula $WBGT = 0.84T_w + 0.30T_g - 0.08T_a$, characterized by a negative coefficient of air temperature T_a , as an alternative to the original outdoor formula $WBGT = 0.7T_w + 0.2T_g + 0.1T_a$. Finally, we indicated the characteristics of the WBGT as an index as well as instructions for use.

INTRODUCTION

The Wet Bulb Globe Temperature (WBGT [1]) has been developed from experiments by Yaglou and Minard in the U.S. in 1957 for the prevention of heat stroke in soldiers. The index is standardized by the National Institute for Occupational Safety and Health (NIOSH) and the International Organization for Standardization (ISO) as ISO-7243. Therefore the index is now frequently used as an evaluation index for thermal environment during work and outdoor sports, and its use is recommended in many countries including Japan.

The WBGT was developed based on experiments with humans, but its background has not been discussed from the perspective of the heat transfer theory. The physical heat exchange between humans and the environment provides physiological and psychological reactions. Hence the properties of indices, even those based on human experiments, can be examined from the viewpoint of physical heat balance.

In this paper, we carry out theoretical derivation and analysis of the WBGT for outdoor conditions employing the heat transfer theory based on the heat balance equation between human body and an outdoor environment. Then, based on the derived theoretical formula, we clarify the structures of the three constant coefficients of wet-bulb temperature, globe temperature and air temperature that define the original WBGT formula for outdoors. In addition, the derived formula for outdoors is compared with that for indoor conditions derived in another paper [2], and we consider points of difference and similarity between two formulae. Furthermore, we perform an examination using actual values to present the characteristics and applicable conditions of the WBGT.

METHODS

Yaglou and Minard have suggested the original WBGT formula (1) for outdoor conditions.

$$\text{WBGT} = 0.7T_w + 0.2T_g + 0.1T_a \quad (1)$$

where WBGT is wet bulb globe temperature [$^{\circ}\text{C}$], T_w is (natural) wet bulb temperature [$^{\circ}\text{C}$], T_g is globe temperature [$^{\circ}\text{C}$] and T_a is air temperature [$^{\circ}\text{C}$].

The heat balance equation (2) between the human body and an outdoor environment is expressed as follows:

$$M = (C + R) + E_{sk} + E_{res} + W + S \quad (2)$$

where M is metabolic rate per unit body surface area [W/m^2], C is convective heat loss [W/m^2], R is radiative heat loss [W/m^2], E_{sk} is evaporative heat loss [W/m^2], E_{res} is convective and evaporative heat loss from respiration [W/m^2], W is external mechanical work [W/m^2] and S is rate of heat storage [W/m^2].

The heat transfer equations by convection C , radiation R [3], evaporation E_{sk} and respiration E_{res} [4] are given as the following equations.

$$C = h_c F_{cl} (T_{sk} - T_a) f_{cl} \quad (4)$$

$$R = h_r F_{cl} [(T_{sk} + 273) - \lambda (T_{gr} + 273)] f_{cl} f_{ref} - F_{cl} (H_d + H_s + H_r) f_{cl} \quad (5)$$

$$E_{sk} = LR h_c F_{pcl} (P_{sk} - P_a) f_{cl} \quad (6)$$

$$E_{res} = 0.0014M(35 - T_a) + 0.0173M(5.87 - P_a) \quad (7)$$

where h_c and h_r are human's convective and linear radiative heat transfer coefficients [$\text{W}/(\text{m}^2\text{C})$], F_{cl} is thermal efficiency factor [N.D.] [5], T_{sk} is mean skin temperature [$^{\circ}\text{C}$], T_a is air temperature [$^{\circ}\text{C}$], f_{cl} is clothing area factor [N.D.] [6], λ is long-wave radiation coefficient [N.D.] [3], T_{gr} is ground temperature [$^{\circ}\text{C}$], f_{ref} is effective radiant area factor [N.D.], H_d , H_s and H_r are direct, scattered and reflective solar radiation absorbed into body surface [W/m^2], LR is Lewis relationship (= 16.5) [$^{\circ}\text{C}/\text{kPa}$], F_{pcl} is permeation efficiency factor [N.D.] [7], P_{sk} is water vapor pressure at skin temperature [kPa] and P_a is water vapor pressure in air [Pa].

λ in Equation (4), physical quantities expressing the properties of radiation specific to the region, date and time, are defined by the following equations:

$$\lambda = (1 + \Omega)^{0.25} (1 - U_r)^{0.25} \quad (7)$$

$$\Omega = (T_{sky} + 273)^4 U_r / (T_{gr} + 273)^4 (1 - U_r) \quad (8)$$

where Ω is ratio of atmospheric radiation to long-wave radiation from the ground to the human body [N.D.], T_{sky} is hypothetical sky temperature [$^{\circ}\text{C}$], U_r is sky view factor for the human body [N.D.].

Also, Ω in Equation (7) expresses a ratio of atmospheric radiation to long-wave radiation from the ground to the human body. U_r in Equations (7) and (8) denote configuration factor between the human body and the sky, and is referred to as sky view factor for the human body [3]. In this study, $f_{ref} = 1$ in equation (4).

Even in the outdoor environment with solar radiation, the wet bulb thermometer is essentially used for measurement in shaded areas, so there is no need to modify the general heat balance equation (9) for the wet bulb [2]. Because the sensing part of the wet bulb thermometer is very small, on the assumption that h_r'/h_c' is nearly equal to zero, Equation (9) leads to Equation (10).

$$h_c'(T_w - T_a) + h_r'(T_w - T_r) + LR h_c'(P_w^* - P_a) = 0 \quad (9)$$

$$(T_w - T_a) + LR(P_w^* - P_a) = 0 \quad (10)$$

where h_c' and h_r' are convective and linear radiative heat transfer coefficients of wet bulb [$\text{W}/(\text{m}^2\text{C})$], T_r is mean radiant temperature [$^{\circ}\text{C}$], P_w^* is saturated water vapor pressure on wet bulb [kPa].

As far as a hot environment surrounding human work is concerned, the relationship of the saturated water vapor pressure P_w^* to the wet bulb temperature T_w on the psychrometric chart

is sufficiently linear, and can be approximated as follows:

$$P_w^* = \kappa T_w + \zeta \quad (11)$$

where κ and ζ are constants of linear approximation of saturated water vapor pressure to wet bulb temperature [kPa/°C].

During work in a hot environment, mean skin temperature is within a range of 35-40°C. Therefore, by treating skin temperature in the same way as the wet bulb temperature, the relationship of saturated water vapor pressure on skin P_{sk} to skin temperature T_{sk} can be linearly approximated by Equation (12).

$$P_{sk}^* = \kappa^* T_{sk} + \zeta^* \quad (12)$$

where κ^* and ζ^* are constants of linear approximation of saturated water vapor pressure to mean skin temperature [kPa/°C].

In the forth coming numerical examination, we will adopt the following values shown in Reference [2]: $\kappa = \kappa^* = 0.279$ kPa/°C and $\zeta = \zeta^* = -4.03$ kPa.

Water vapor pressure on skin surface P_{sk} is expressed by Equation (12)', accompanied with μ that indicates the degree of saturation on skin surface. Here we expand equations with μ preserved, but in the later stages, we will suppose $\mu = 1$ because the WBGT index is designed for use within thermal limitation where water vapor pressure on skin surface is almost saturated in consequence of promoted sweating.

$$P_{sk} = \mu (\kappa^* T_{sk} + \zeta^*) \quad (12)'$$

where μ is saturated ratio of water vapor pressure to skin temperature [N.D.].

The globe thermometer is basically intended for measuring heat radiation, and steady-state heat balance after sufficient exposure of the globe thermometer to an outdoor environment with solar radiation for actual measurement is expressed, with reference to heat transfer equation by radiation for the human body (4), as follows:

$$h_c''(T_g - T_a) + h_r'' F_{cl}'' [(T_g + 273) - \lambda''(T_{gr} + 273)] f_{cl}'' - F_{cl}'' (H_d'' + H_s'' + H_r'') f_{cl}'' = 0 \quad (13)$$

where h_c'' and h_r'' are convective and linear radiative heat transfer coefficient of globe [W/(m²°C)], λ'' is long-wave radiation coefficient of globe [N.D.], F_{cl}'' is thermal efficiency factor of globe [N.D.], f_{cl} is clothing area factor of globe [N.D.], H_d'' , H_s'' and H_r'' are direct, scattered and reflected solar radiation absorbed into globe [W/m²].

For the globe thermometer, $F_{cl}''=1$ and $f_{cl}''=1$. In this study, $\lambda''=\lambda$.

RESULTS

Substituting the relevant equations including Equations (3) ~ (8) and (10) ~ (13) into heat balance equation of the human body (2) gives the outdoor WBGT formula (14) in conformity with the indoor WBGT formula [2].

$$\begin{aligned} & \left[(h_c + h_r) F_{cl} f_{cl} + \mu \kappa^* LR h_c F_{pcl} f_{cl} \right] T_{sk} + \left[F_{cl} f_{cl} (H_d'' + H_s'' + H_r'') \frac{\lambda h_r}{\lambda'' h_r''} - F_{cl} f_{cl} (H_d + H_s + H_r) \right. \\ & \left. + (\mu \zeta^* - \zeta) LR h_c F_{pcl} f_{cl} - M(0.8494 + 0.0173\zeta) + W + S \right] \\ & = \left[h_c F_{pcl} f_{cl} (1 + \kappa \cdot LR) + 0.0173M \left(\kappa + \frac{1}{LR} \right) \right] T_w \\ & + \left[h_r F_{cl} f_{cl} \frac{\lambda}{\lambda''} \left(1 + \frac{h_c''}{h_r''} \right) \right] T_g \\ & + \left[F_{cl} f_{cl} \left(h_c - h_c'' \frac{\lambda h_r}{\lambda'' h_r''} \right) - h_c F_{pcl} f_{cl} + M \left(0.0014 - \frac{0.0173}{LR} \right) \right] T_a \end{aligned} \quad (14)$$

Writing the coefficient of T_{sk} and the constant term on the left-hand side of Equation (14) as ξ and η , respectively, and the coefficients of T_w , T_g and T_a as α , β and γ , we obtain equations (15) ~ (20).

$$\left[(h_c + h_r)F_{cl}f_{cl} + \mu\kappa^*LRh_cF_{pcl}f_{cl} \right] \equiv \xi \quad (15)$$

$$\left[F_{cl}f_{cl}(H_d'' + H_s'' + H_r'')\frac{\lambda h_r''}{\lambda'' h_r''} - F_{cl}f_{cl}(H_d + H_s + H_r) \right. \\ \left. + (\mu\zeta^* - \zeta)LRh_cF_{pcl}f_{cl} - M(0.8494 + 0.0173\zeta) + W + S \right] \equiv \eta \quad (16)$$

$$\left[h_cF_{pcl}f_{cl}(1 + \kappa \cdot LR) + 0.0173M\left(\kappa + \frac{1}{LR}\right) \right] \equiv \alpha \quad (17)$$

$$\left[h_rF_{cl}f_{cl}\frac{\lambda}{\lambda''}\left(1 + \frac{h_c''}{h_r''}\right) \right] \equiv \beta \quad (18)$$

$$\left[F_{cl}f_{cl}\left(h_c - h_c''\frac{\lambda h_r''}{\lambda'' h_r''}\right) - h_cF_{pcl}f_{cl} + M\left(0.0014 - \frac{0.0173}{LR}\right) \right] \equiv \gamma \quad (19)$$

$$\xi T_{sk} + \eta = \alpha T_w + \beta T_g + \gamma T_a \quad (20)$$

Dividing both sides of Equation (20) by ξ yields Equation (21), a form based on the skin temperature T_{sk} .

$$T_{sk} + \left(\frac{\eta}{\xi}\right) = \left(\frac{\alpha}{\xi}\right)T_w + \left(\frac{\beta}{\xi}\right)T_g + \left(\frac{\gamma}{\xi}\right)T_a \quad (21)$$

Equation (21), on removing the physical quantity related to solar radiation, corresponds to the WBGT formula for indoor use [2].

We examine the varying properties of the variable coefficients in Equation (21) that correspond to the constant coefficients in the original WBGT formula (1) for outdoor conditions with solar radiation, substituting concrete values. We calculated the value of η/ξ , α/ξ , β/ξ and γ/ξ , changing metabolic rate from 1 to 4 met, clo unit from 0.2 to 1.0 clo and outdoor air velocity from 1.0 to 5.0 m/s with global solar radiation set at 600 W/m².

The calculated values of η/ξ ranged from -10.0 to 2.3. If air velocity, clo unit and metabolic rate were low, the η/ξ value was positive.

Figures 1 to 3 show the results of calculation of the α/ξ , δ/ξ and γ/ξ values. Figure 1 shows the variation of each coefficient in response to the variation of metabolic rate M with clo unit and air velocity held constant. The horizontal axis indicates metabolic rate, and the vertical axis indicates the α/ξ , β/ξ , γ/ξ and $\alpha/\xi + \beta/\xi + \gamma/\xi$ values. Responding to metabolic rates of 1 to 4 met, α/ξ ranges from 0.81 to 0.86, β/ξ is constant at 0.33 and γ/ξ is approximately -0.12. Thus, the values of α/ξ , β/ξ and γ/ξ are roughly 0.84, 0.33 and -0.12 rather than 0.7, 0.2 and 0.1, the coefficients in the original formula (1). The sum of α/ξ , β/ξ and γ/ξ ranges from 1.02 to 1.07, slightly exceeding 1.0. It is remarkable in the numerical examination in Figure 1, also in the following examinations in Figure 2 and 3, that the mean value of the coefficient γ/ξ of T_a is negative. The value shifts to the positive side when air velocity exceeds a certain value.

Figure 2 shows the variation of the coefficients in response to the variation of air velocity (horizontal axis) with metabolic rate and clo unit held constant. The values of α/ξ , β/ξ and γ/ξ are, respectively, 0.84 ~ 0.82, 0.33 ~ 0.24 and -0.12 ~ -0.02, as in Figure 1. In Figure 2, unlike Figure 1, the α/ξ and β/ξ values decrease, when the sum of the three coefficients approaches 1.0.

Similarly, Figure 3 shows the variation of the coefficients in response to the variation of clo unit with metabolic rate and air velocity held constant. The values of α/ξ , β/ξ and γ/ξ are 0.86

~ 0.84, 0.24 ~ 0.29 and -0.08 ~ -0.05, respectively. As clothing insulation increases, the α/ξ values decreases and the β/ξ and γ/ξ values increase. The sum of α/ξ , β/ξ and γ/ξ increases gradually from 1.02 to 1.07.

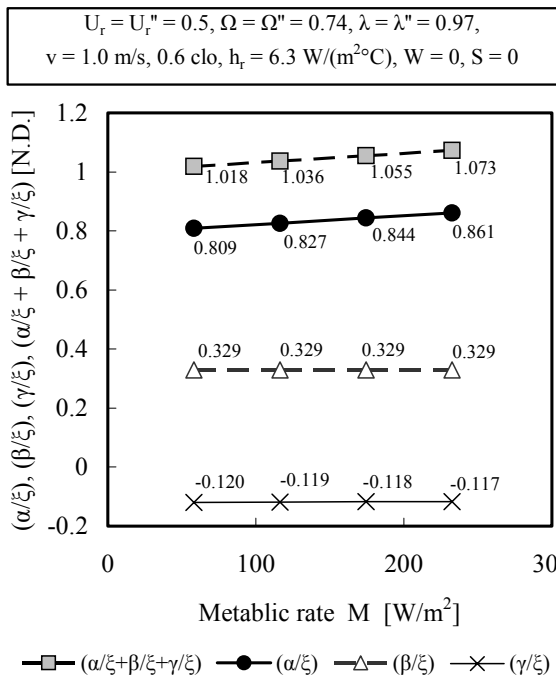


Figure 1 The variation of each coefficient in eq.(21) in case of variable metabolic rate with constant clo unit and air velocity.

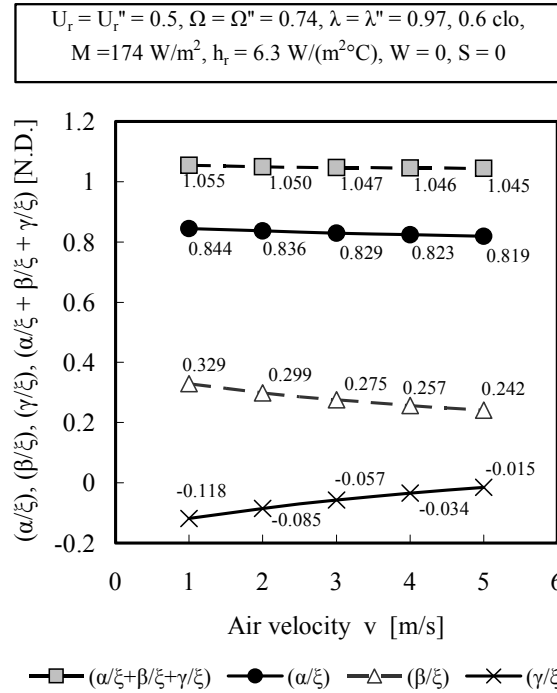


Figure 2 The variation of each coefficient in eq.(21) in case of variable air velocity with constant clo unit and metabolic rate.

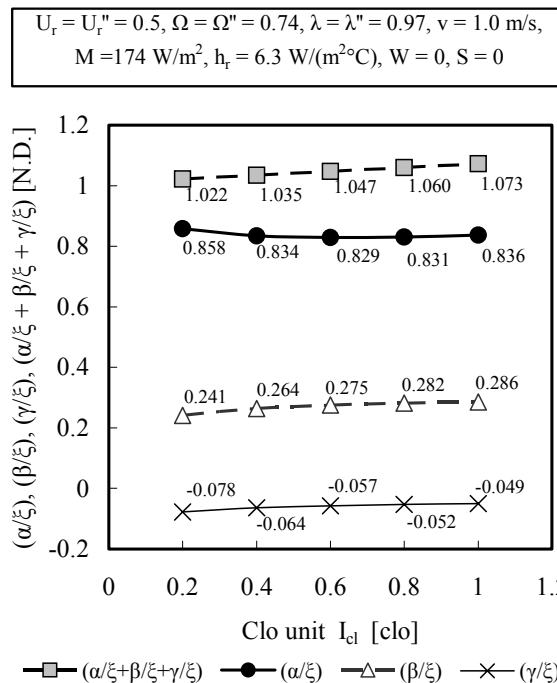


Figure 3 The variation of each coefficient in eq.(21) in case of variable clo unit with constant metabolic rate unit and air velocity.

DISCUSSION

We consider points of difference and similarity between the derived formula for outdoor conditions and that for indoor conditions [2]. The derived formula for indoor conditions in Reference [2] is shown as follows:

$$\begin{aligned}
 & \left[(h_c + h_r) F_{cl} f_{cl} + \mu \kappa^* \cdot LR h_c F_{pcl} f_{cl} \right] T_{sk} \\
 & + \left[(\mu \zeta^* - \zeta) LR \cdot h_c F_{pcl} f_{cl} - M(0.8494 + 0.0173\zeta) + W + S \right] \\
 & = \left[h_c F_{pcl} f_{cl} (1 + \kappa \cdot LR) + 0.0173M \left(\kappa + \frac{1}{LR} \right) \right] T_w \\
 & + \left[F_{cl} f_{cl} (h_c'' + h_r) \right] T_g \\
 & + \left[F_{cl} f_{cl} (h_c - h_c'') - h_c F_{pcl} f_{cl} + M \left(0.0014 - \frac{0.0173}{LR} \right) \right] T_a
 \end{aligned} \tag{22}$$

Comparing the coefficients of T_w , T_g and T_a in Equation (14) with those in Equation (22), the coefficient of first term on the left- and right-hand side is the same between two equations. The coefficient of second term on the left-hand side is the same constituent elements between two equations except solar radiation term. Meanwhile, the second term on the right-hand side in equation (14) includes the long-wave radiation coefficient, and third term in equation (14) includes the elements with respect to globe thermometer. Other elements of the coefficients in two equations are equally common. Examination using concrete values in Equation (22) provides that coefficient values in two equations were nearly equal.

Examination of the derived formula confirmed that the coefficients of T_w , T_g and T_a in the derived WBGT formula, considering their structures, are not strictly constant but variable depending on metabolic rate, clothing insulation, air velocity and other such factors.

We calculated the concrete values of each coefficient in Equation (14), changing metabolic rate, clothing insulation and air velocity. The calculated values of η/ξ ranged from -10.0 to 2.3 . We consider the effect of solar radiation on the η/ξ qualitatively. Assuming that $\mu = 1$ in Equation (16), the inequality, which turns into $\eta \geq 0$, is expressed as follows:

$$\frac{F_{cl} f_{cl} \left[(H_d'' + H_s'' + H_r'') - (H_d + H_s + H_r) \right] + W + S}{M} \geq 0.781 \tag{23}$$

Equation (23) shows that $\eta \geq 0$ is obtained if the sum of the solar radiation H , external work W and heat storage S for the metabolic rate M is greater than 0.781 . The difference between the first and second terms in the left-hand side of Equation (23) represents the difference between solar radiation absorbed by the globe thermometer and that absorbed by the human body. This difference, dependent on solar absorptance, projected area factor and configuration factor, is positive because solar absorptance of the human body is generally less than that of the globe thermometer. This indicates that in an outdoor environment with solar radiation, the η/ξ value may be positive even without external work or heat accumulation, and also that in an environment without solar radiation, the η/ξ value cannot be positive without them.

The calculation showed the coefficients α/ξ , β/ξ and γ/ξ of T_w , T_g and T_a to be $0.81 \sim 0.86$, $0.25 \sim 0.33$ and $-0.12 \sim -0.02$, respectively, roughly 0.84 , 0.30 and -0.08 rather than 0.7 , 0.2 and 0.1 the coefficients in the original formula. The coefficient γ/ξ of T_a is negative value for low air velocities and positive for high air velocities, and that lower air velocities resulted in larger negative values. Negative values, seen for low air velocities, mean a relatively small influx of dry heat by convection and radiation from an environment and the cooling of skin due to sweat evaporation, whereas positive values, seen for high air velocities, indicate a relatively large influx of dry heat from the environment which, negating the effect of sweat evaporation, raises the skin temperature T_{sk} .

