ANALYSIS OF CHAOTIC BEHAVIOR OF INDOOR RADON CONCENTRATIONS

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Indoor 222Rn concentrations are influenced by several factors which may change with time, thereby causing temporal fluctuations of radon concentrations in rooms. Different chaos based measurements (fractal analyses) were applied to radon time series in three different rooms (kitchen, working room, radiation laboratory) in Austria to determine the degree of chaotic behavior, to predict fluctuations in the future, and to investigate correlation's with meteorological parameters.

Our fractal analyses of these indoor radon time series demonstrated that indoor 222Rn concentrations do indeed exhibit features which are characteristic of chaotic systems. The computed fractal dimensions, such as Hurst exponent, Lyapunov exponent, capacity dimension, embedding dimension and attractor dimension, provided estimates of the degree of chaotic behavior, such as low dimensional chaos for the kitchen, high dimensional chaos for the working room and chaos with additive noise for the radiation laboratory. Application of a nonlinear prediction algorithm revealed that the predictability of radon time series is restricted to approximately three relative time steps into the future and into the past, e.g. monthly radon measurements cannot be reliably extrapolated beyond a period of three months. If the similarity of fractal dimensions between radon concentrations and meteorological parameters is used as a measure of the degree of correlation, our analyses suggest a strong correlation between radon concentrations and negative pressure differences.

Key words: radon, indoor, meteorological parameters, fractal analysis, chaotic systems

INTRODUCTION

To understand the dynamics of indoor radon concentrations, it is common practice to measure radon concentrations continuously over some period of time. Such radon measurements can be used to derive statistically significant average concentrations or to study their dependence on various factors, such as ventilation conditions. In the present study, radon time series were analyzed (i) to determine the degree of chaotic behavior by fractal analysis, (ii) to predict their fluctuations in the future by a nonlinear prediction algorithm based on fractal methods, and (iii) to investigate their correlation's with meteorological parameters by comparison with their fractal dimensions.

Indoor 222Rn concentrations are determined by several factors which may change with time, and, in turn, cause temporal fluctuations of the radon concentrations in rooms. In a deterministic system, the causal relationship between the radon concentration and a given controlling parameter yields total correlation between radon concentrations measured at two adjacent time steps, while no correlation at all exists in a purely stochastic (or random) system. The intermediate chaotic system is characterized by its determinism (attractor), superimposed by random variations, i.e., there is a statistical correlation between two consecutive radon measurements. Thus, from a modeling perspective, a chaotic system is equivalent to a stochastic system constrained by correlation's.
The application of fractal methods to radon time series has previously been demonstrated by Cuculeanu et al. (1996), who determined the fractal properties of $^{222}\text{Rn}$ and $^{220}\text{Rn}$ outdoor time series using Hurst’s rescaled range analysis and the box counting method. In the present study, fractal analyses of indoor $^{222}\text{Rn}$ time series were performed using different chaos theory based measurements: (i) the time delay method (embedding dimension $E$, attractor (fractal) dimension $D$), (ii) Hurst’s rescaled range analysis (Hurst exponent $H$, attractor (fractal) dimension), (iii) the Lyapunov exponent $L$ (entropy $S$), (iv) the capacity (Hausdorff, box-counting or fractal) dimension, $D_C$, and (v) a nonlinear prediction algorithm (embedding dimension $E$, attractor dimension $D$). For most of the calculations presented here, the Chaos Data Analyzer (CDA 2.0) (Sprott and Rowlands 1995) was used together with the Signal ANalysis and TJme Series processing algorithm (SANTIS 1.1) (Vandenhouten et al. 1995).

The above described fractal methods were applied to three $^{222}\text{Rn}$ indoor time series from three different locations in Austria, a kitchen in Straßwalchen (Salzburg), a working room in Traun (Upper Austria), and a radiation laboratory at the Institute of Physics and Biophysics in Salzburg.

FRACTAL METHODS

The application of fractal methods allows us to explore the chaotic nature of indoor radon concentrations. It must be noted, however, that different chaos based measurements provide different measures of the degree of chaotic behavior. The salient features of the fractal techniques employed in the present study will briefly be described below.

Time delay method

From the original time series, an embedding space of dimension $E$ can be constructed, using the method of time series embedding with time delays (Takens 1981) (note: the embedding dimension was also determined by the nonlinear prediction algorithm described later). The embedding dimension $E$ can be interpreted as the number of degrees of freedom in a dynamic system, i.e., the greater the embedding dimension, the more chaotic is the system. More importantly, however, the attractor (fractal) dimension $D$ can be derived from the relation

$$E_{\text{min}} < 2D + 1$$

where $E_{\text{min}}$ is the minimal embedding dimension. Values of the attractor dimension $D$ between 2 and 5 are a strong indicator of chaotic behavior.

Rescaled range analysis (Hurst exponent)

The Rescaled Range Analysis $R/s$ (range/standard deviation) developed by Hurst et al. (1965) provides a simple tool for analyzing time series in the form of a so-called Hurst plot. The Hurst exponent $H$, which ranges between 0 and 1, can be derived as the slope in the Hurst plot, in which log ($R/s$) is plotted against log $\tau$, where $\tau$ is the time step. While a value of 0.5 represents a true random walk, a value of $0.5 < H < 1$ indicates a so-called persistent behavior (i.e., one can expect with increasing certainty as the value moves toward 1 that whatever direction of change has been current will continue). Similarly, values of $0 < H < 0.5$ indicate anti-persistent behavior. The Hurst exponent $H$ can also be used to compute the attractor (fractal) dimension $D$ from the relationship

$$E_{\text{min}} < 2D + 1$$

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Feder (1988) has shown that a linear fit to the Hurst plot underestimates the slope $H$ and hence overestimates the fractal dimension in equation (2), and vice versa. This is part of the reason that measured Hurst dimensions do not necessarily agree numerically with other fractal measurement tools, although they remain an extremely efficient tool for comparing one data set to another (Russ 1994).

**Lyapunov exponent**

The Lyapunov exponent $L$ illustrates the system's Sensitivity on Initial Conditions (SIC). (Wolf, 1985, Feder 1988). Positive Lyapunov exponents are considered evidence of chaos, while negative exponents suggest a mean reverting behavior. The larger the positive exponent, the more chaotic is the system and, conversely, the shorter is the time scale of the system's predictability. Since there are as many exponents as there are dynamical equations, only the most positive exponent is calculated here. The sum of the positive Lyapunov exponents (base $e$) is called entropy $S$; its reciprocal is roughly the time over which meaningful prediction is possible (Sprott and Rowlands 1995).

**Capacity (fractal) dimension**

A fractal is by definition a set for which the Hausdorff dimension strictly exceeds the topological dimension; hence it is lower than the Euclidian dimension (Mandelbrot 1982). Since the calculation of the Hausdorff dimension is very complicated, the similar capacity dimension (sometimes also called box-counting or fractal dimension) is used instead.

The capacity dimension $D_C$ is calculated by successively dividing the phase space with embedding dimension $D$ into equal hypercubes and plotting the log of the fraction of hypercubes that are occupied with data points versus the log of the normalized linear dimension of the hypercubes (Sprott and Rowlands 1995). Values of the capacity dimension between 2 and 5 indicate chaotic behavior, consistent with the attractor dimension.

**Nonlinear forecasting algorithm**

An essential property of chaos is its determinism, i.e., chaotic systems obey certain rules. Trajectories of chaotic systems can be predicted for short time scales. However, chaos amplifies noise exponentially and, as a result of this, short term determinism becomes long-term randomness. The limited predictive power of chaotic dynamical systems is because they are Sensitive to Initial Conditions (SIC) and because we cannot have perfect measurements (which require an infinite amount of information). One would therefore expect chaos to be characterized by a decrease of the correlation between predicted and actual values as prediction time increases.

In addition, the nonlinear prediction algorithm can also be used to calculate the embedding dimension $D$ and, subsequently, the attractor dimension $D$, in a way similar to the method proposed by Sugihara and May (1990). This method is quite simple, but it is also robust to the effect of noise in the data which can cause problems with more sophisticated methods.
FRACTAL ANALYSES OF \(^{222}\text{Rn}\) CONCENTRATION TIME SERIES

The above described fractal methods were applied to three indoor \(^{222}\text{Rn}\) time series (consisting of N measurements) from three different locations in Austria. Two of the rooms, a kitchen in Straßwalchen (N = 2275 in 10 minutes intervals) in spring of 1997, with an average radon concentration of 30 Bq m\(^{-3}\), and a working room in Traun (N = 3442 in 1 hour intervals) in spring of 1998, with an average radon concentration of 415 Bq m\(^{-3}\), were in brick built private houses. The third room, a radiation laboratory at the Institute of Physics and Biophysics in Salzburg (N = 6867 in 10 minutes intervals) in spring of 1998, with an average radon concentration of 16 Bq m\(^{-3}\), is housed in a concrete building. All radon measurements were made with an AlphaGuard PQ 2000.

To illustrate the nature of the analyzed data, the radon concentrations measured in the kitchen in Straßwalchen are shown in Figure 1. All fractal methods applied to this data set produce fractal dimensions which clearly demonstrate the existence of chaotic behavior. For example, taking the the optimum embedding dimension E to be that yielding the highest correlation coefficient, it can be seen from Figure 2 that the highest correlation coefficient was found for E = 5 – 7. According to equation (1) this embedding dimension yields an attractor dimension D = 2 – 3. The length of the plateau further indicates the existence of low dimensional chaos (Sugihara and May 1990).

The results of our fractal analyses of radon time series measured in the three rooms are compiled in Table 1. The values of the different fractal measures demonstrate that indoor radon concentrations in all three rooms exhibit the features of chaotic systems. All predicted fractal measurement values are associated with an error tolerance, which is defined as 2.5 times the standard deviation of the slope in the various plots, divided by the square root of the number of trajectories analyzed (Sprott and Rowlands 1995). These error tolerances are not statistically significant in a mathematical sense; rather they serve as a warning against misplaced trust in the exact numerical values.

While the radon data in two rooms (kitchen, radiation laboratory) were recorded every 10 minutes, the radon concentrations in the working room were measured in 60 minutes intervals. To check the effect of the length of the time steps on the results of the fractal analyses, the 10 minutes data were converted into average 1 hour values and then analyzed again. The obtained fractal dimensions for the modified data set were not different from those derived from the original data set, which is consistent with the findings of Sugihara and May (1990).

While the fractal parameters computed for the kitchen indicate the presence of low dimensional deterministic chaos, the working room is characterized by high dimensional chaos. In case of high dimensional chaos it is not possible to confirm the attractor dimension calculated by the nonlinear prediction algorithm by other fractal methods, such as the Hurst exponent or the capacity (fractal) dimension (Table 1).

In case of the radiation laboratory, it was not easy to define a proper embedding dimension E due to the lack of a distinct plateau. Together with the relatively large Hurst exponent, this is a strong indication that this chaotic time series is “contaminated“ in some way, either by systematic changes (such as periodical ventilation) or by strong additive noise. Consequently, the fractal measurements for the laboratory data suggest the presence of chaos superimposed by some noise (Sugihara and May 1990).
PREDICTION OF FUTURE BEHAVIOR

The limited predictive power of chaotic systems is caused by their sensitivity to initial conditions and the limited precision of any measurement. As a result of this, trajectories of chaotic systems can only be predicted for relatively short time scales. Using a nonlinear prediction algorithm (Sprott and Rowlands 1995), predictions were generated by cutting the last eighteen measurements and then predicting them on the basis of the remaining measurements, using them as a library of past patterns.

As an example, predictions of the radon time series in the kitchen are displayed in Figure 3. The points connected by solid lines represent the prediction errors as a function of the prediction time (in relative time steps) for an optimum embedding dimension E = 6. The overall decline in prediction accuracy, illustrated by a growing prediction error with increasing time into the future, is an unmistakeable feature of chaotic dynamics.

From the sum of the positive Lyapunov exponents (base e), we calculated an entropy S of 0.293. Since the reciprocal of the entropy is roughly the time over which a prediction is meaningful (Sprott and Rowlands 1995), a value of $1/S = 3.41$ suggests that predictions of the future behaviour of indoor radon concentrations are only possible for a period of about three relative time steps, e.g. approximately three hours for hourly measurements or three days for daily measurements. The entropies and the related times of predictability for the three rooms are listed in Table 2.

FRACTAL ANALYSES OF METEOROLOGICAL PARAMETERS

Temporal variations of indoor radon concentrations in closed rooms are caused primarily by changes in meteorological conditions (Steinhäusler 1975). Temperature and barometric pressure were measured simultaneously in each room and these measurements (and related quantities) were analyzed by the same fractal methods as above applied to the radon concentrations. In the present study it is assumed that the measure of the degree of correlation between these meteorological parameters and their corresponding radon concentrations is the similarity of their fractal dimensions.

For example, negative pressure differences in the kitchen are plotted in Figure 4 as a function of the number of relative time steps (in 10 minutes intervals). The resulting embedding dimension plot is presented in Figure 5, yielding an embedding dimension in the range of 5 – 8 which is very similar to the embedding dimension found for the corresponding radon concentrations. This a clear indication that negative pressure differences act in the same chaotic way as the radon time series. Indeed, the similarity of all fractal parameters compiled in Table 3 suggests that negative pressure differences and indoor radon concentrations in the kitchen are strongly correlated. While such a relationship has also been observed for the other two rooms, practically no correlation could be found with temperature and pressure in any of the three rooms.

CONCLUSIONS

The fractal analyses described in the present study represent a novel approach for studying $^{222}$Rn time series. Analysis of chaotic time series has greatly enhanced the understanding of chaos in experimental systems by allowing multi-dimensional dynamical information to be recovered from a time series of measurements of a single variable. Thus the strange attractor of a chaotic system can
often be extracted from a time series of measurements of a single variable, in our case the radon concentration (Giroletti 1991).

Fractal analyses of $^{222}\text{Rn}$ time series allow to distinguish between chaotic behavior (deterministic chaos) and noise (random fluctuations). This information may be used to understand the physical mechanisms behind the chaotic behavior, such as the dependence of indoor radon concentrations on meteorological factors or ventilation conditions.

The forecasting technique discussed here is phenomenological in a sense that it attempts to assess the qualitative character of a system's dynamics. It is possible to make short-range predictions based on that premise without attempting to provide an understanding of the physical mechanisms that govern the behavior of the system.

The following conclusions can be drawn from our fractal analyses of indoor radon time series:

1. Indoor $^{222}\text{Rn}$ concentrations do indeed exhibit features which are characteristic of chaotic systems.
2. Fractal dimensions, such as Hurst exponent, Lyapunov exponent, capacity dimension, embedding dimension and attractor dimension, provide estimates of the degree of chaotic behavior. For example, our analyses of radon time series suggest low dimensional chaos for the kitchen, high dimensional chaos for the working room and chaos with additive noise for the radiation laboratory.
3. The predictability of radon time series is restricted to approximately three relative time steps into the future and into the past. For example, integrated radon measurements over a one month period cannot be reliably extrapolated beyond a period of three months.
4. The similarity of fractal dimensions between radon concentrations and meteorological parameters provides a measure of their degree of correlation. For example, the apparent similarity of fractal dimensions for radon concentrations and negative pressure differences suggests a strong correlation between the two parameters.

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<table>
<thead>
<tr>
<th>Site</th>
<th>Hurst exponent</th>
<th>Lyapunov exponent</th>
<th>Capacity dimension</th>
<th>Embedding dimension</th>
<th>Attractor dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kitchen (Strasswalchen)</td>
<td>0.19 (± 0.04)</td>
<td>0.34 (± 0.03)</td>
<td>2.35 (± 0.35)</td>
<td>5 – 7</td>
<td>2 – 3&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Working room (Traun)</td>
<td>0.38 (± 0.08)</td>
<td>0.21 (± 0.01)</td>
<td>c</td>
<td>8 – 10</td>
<td>5 – 6</td>
</tr>
<tr>
<td>Laboratory (Salzburg)</td>
<td>0.02 (± 0.01)</td>
<td>0.52 (± 0.03)</td>
<td>2.69 (± 0.23)</td>
<td>5 – 6</td>
<td>1.5 – 2</td>
</tr>
</tbody>
</table>

<sup>a</sup> determined by a non-linear prediction algorithm

<sup>b</sup> 2.5 times the standard deviation divided by the square root of the number of trajectories analysed

<sup>c</sup> not possible to determine
Table 2. Values of the entropy $S$ (defined as the sum of the positive Lyapunov coefficients) for the three radon time series analyzed. The number of relative time steps over which a prediction is meaningful is given by $1/S$.

<table>
<thead>
<tr>
<th>Site</th>
<th>Entropy</th>
<th>Number of relative time steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kitchen (Straßwalchen)</td>
<td>0.293 (± 0.021)</td>
<td>3.41</td>
</tr>
<tr>
<td>Working room (Traun)</td>
<td>0.457 (± 0.018)</td>
<td>2.19</td>
</tr>
<tr>
<td>Laboratory (Salzburg)</td>
<td>0.512 (± 0.034)</td>
<td>1.95</td>
</tr>
</tbody>
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Table 3. Comparison of fractal parameters derived from the continuous measurements of radon concentrations and barometric pressures in a kitchen in Straßwalchen (Austria).

<table>
<thead>
<tr>
<th>Fractal parameter</th>
<th>$^{222}$Rn concentration</th>
<th>Negative pressure difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hurst exponent</td>
<td>0.19</td>
<td>0.07</td>
</tr>
<tr>
<td>Lyapunov exponent</td>
<td>0.34</td>
<td>0.17</td>
</tr>
<tr>
<td>Capacity dimension</td>
<td>2.35</td>
<td>1.72</td>
</tr>
<tr>
<td>Embedding dimension</td>
<td>5 - 7</td>
<td>5 - 8</td>
</tr>
<tr>
<td>Attractor dimension</td>
<td>2 - 3</td>
<td>2 - 2.5</td>
</tr>
</tbody>
</table>
Figure 1. $^{222}$Rn concentration time series obtained in a kitchen in Straßwalchen (Austria), based on 2275 measurements in 10 minutes intervals. The average radon concentration is 30 Bq m$^{-3}$. 
Figure 2. Determination of the embedding dimension for the radon measurements in a kitchen in Straßwalchen (Austria) by calculating the correlation between the last eighteen predicted and observed values for different embedding dimensions.
Figure 3. Correlation between prediction error and prediction time (in relative time steps) with embedding dimension $E = 6$ for the radon concentrations measured in a kitchen in Straßwalchen (Austria).
Figure 4. Negative pressure differences as a function of time (in 10 minutes intervals) in a kitchen in Straßwalchen (Austria). The simultaneously measured radon concentrations are displayed in Figure 1.
Figure 5. Calculation of the embedding dimension for the negative pressure difference in a kitchen in Straßwalchen (Austria).