Reduced order model for air temperature control in indoor spaces

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ABSTRACT

Real-time control of comfort in indoor spaces needs models of temperature distribution and air-velocity velocity field. Complete models, based on CFD, give this information but are improper for real-time calculations. Therefore, a reduced model is needed. This study proposes to reduce the dimension of a CFD model by first considering the velocity field fixed and solving only the energy balance equation, then putting this equation in the form of state-space and finally by reducing its order by Proper Orthogonal Decomposition (POD). This algorithm was applied to a room equipped with a fan coil with an air jet having three patterns: sticking to the ceiling and reaching the opposite wall, falling before the opposite wall, and falling before reaching the ceiling. For the two-dimensional case, the reduced model was validated by comparison with CFD results for step inputs of temperature and air velocity. As the order of the reduced model is always smaller than 7, the energy balance equation may be solved in real time and integrated into a control algorithm.

1 INTRODUCTION

In air conditioning spaces, the air velocity field and the temperature distribution are not uniform, with implications on thermal comfort and energy consumption. Generally, this characteristic is not taken into account by the control system. Even though computational fluid dynamics (CFD) may predict the temperature distribution and the velocity field, computing time is prohibitive for real-time control. For the problem of indoor air flow, Peng (1996) proposed to calculate the dynamic temperature distribution in a fixed flow field, provided that it is correctly calculated by the CFD code. So in this paper, only the energy balance equation needs to be solved. We propose the reduction of this equation by using Proper Orthogonal Decomposition.

2 THEORY

2.1 Energy balance equation in state space

The energy balance is described by the differential equation:

\[
\frac{\partial \theta}{\partial t} + \mathbf{V} \cdot \nabla \theta = \text{div}\left(\rho \mathbf{a}_{\text{eff}} \nabla \theta\right) + \frac{S_p}{\rho C_p}
\]

(1)

where:
- \(\theta\): temperature,
- \(\mathbf{V}\): velocity field,
- \(\rho\): density,
- \(C_p\): constant-pressure specific heat,
- \(S_p\): source term.

Usually, the airflow in rooms is fully turbulent. To treat the turbulence, Reynolds averaging may be used. Consequently, in equation (1), the effective diffusivity, \(a_{\text{eff}}\), can be split into a laminar term and a turbulent term:

\[
a_{\text{eff}} = \frac{\lambda}{\rho C_p} + \frac{\nu_t}{\sigma_t}
\]

(2)

where:
- \(\lambda\): thermal conductivity,
- \(\nu_t\): turbulent viscosity,
- \(\sigma_t\): turbulent Prandtl number, here equal to 0.9.

The velocity field \(\mathbf{V}\) and the turbulent viscosity field are calculated with a CFD software. The computed temperature distribution is considered as the reference and the initial condition. For this study, we used the high Reynolds number k-\(\epsilon\) model for turbulence modeling, Lu et al. (1997), and the robust scheme MARS for space discretization of the convective term STARCD (1999). To discretize equation 1, the mesh and the method of space discretization adopted are the same as in the CFD software; the method used in our study is the control volume (Murakami (1990). For the discretization of the convective term, the widely applied QUICK scheme was chosen. It has a good stability and a relatively small numerical viscosity. Near the wall, convective heat transfer coefficients are extracted from the CFD results. At this stage, the order of the system corresponds to the number of discretization cells, and the form of the discretized equation is:

\[
\frac{d \theta}{dt} = a_\theta \theta + a_{\theta \theta} \theta \theta + a_{\theta \theta \theta} \theta \theta \theta + a_{\theta \theta \theta \theta} \theta \theta \theta \theta + a_{\theta \theta \theta \theta \theta} \theta \theta \theta \theta \theta + a_{\theta \theta \theta \theta \theta \theta} \theta \theta \theta \theta \theta + a_{\theta \theta \theta \theta \theta \theta \theta} \theta \theta \theta \theta \theta + a_{\theta \theta \theta \theta \theta \theta \theta \theta} \theta \theta \theta \theta \theta
\]

(3)
where:
E, W, H, L, N, S: index for East, West, High, Low, North and South neighbouring cells
EE, WW, HH, LL, NN, SS: index for East, West, High, Low, North and South neighbouring cells of respective-
ly E, W, H, L, N, S cells

A more advantageous representation for control is the
state space form in which the temperatures are written
in a vector, Ghiaus and Ghiaus (1999). The high order
model is then:

\[
\begin{aligned}
\dot{\theta} &= A\theta + Bu \\
\dot{y} &= C\theta + Du
\end{aligned}
\]  

(4)

where:
\( \theta \): state vector
\( \dot{\theta} = \frac{d\theta}{dt} \)
\( u \): input vector
\( y \): output vector.

2.2 Reduced order model

The order of equation (3) is too high to be used in
real-time applications. A reduction of the order can be
achieved by finding an optimal basis with the Proper
Orthogonal Decomposition (POD), Allery et al. (2005)
and Gunes (2002). This method needs snapshots ex-
tracted from a transient simulation made with the CFD.

The basic idea of the POD consists in finding a “physi-
cal” basis which is optimal in an energetic sense. Thus,
we search a deterministic function, \( \phi \), which gives the
“best” representation of the set of temperature \( \theta \) (as-
sumed random) in the following sense:

\[
\begin{aligned}
\mathbf{E}(\theta, \phi^2) &= \max_{\nu \in L^2(\Omega)} \left\{ \mathbf{E}(\theta, \nu^2) \right\} \\
(\phi, \phi) &= 1
\end{aligned}
\]  

(5)

where:

\( L^2 : \) space of functions of finite energy in the flow vol-
ume \( \Omega \).

\( (\star, \star) : \) inner product of \( L^2(\Omega) \)

\( \mathbf{E} : \) statistic average operator

From variational, calculus it follows that the above ex-
pression is equivalent to the Fredholm Integral:

\[
\text{Find } \lambda \in \mathbb{R} \text{ and } \phi \in L^2(\Omega)
\]

with

\[
\int_\Omega S(x, x') \phi(x') dx' = \lambda \phi(x)
\]  

(6)

representing an eigenvalue problem for \( \phi \), where \( S \) is
the spatial correlation tensor defined by:

\[
S(x, x') = \mathbf{E}(\theta(x, t) \otimes \theta(x', t))
\]  

(7)

The eigenfunctions \( \phi_n \) are orthogonal and all realiza-
tions of the temperature \( \theta \) are written as:

\[
\theta(x, t) = \sum a_i(t) \phi_i(x) \text{ in } L^2(\Omega) \text{ sense},
\]

with

\[
a_i(t) = (\theta(x, t), \phi_i)
\]  

(8)

In practice, if the sampling of the temperature is ob-
tained by numerical simulation, the evaluation of the
tensor \( S \) is a very huge computational task. In order to
reduce the calculation, we used the snapshots method
proposed by Sirovich (1987). In this technique, based
on the fact that the eigenfunctions can be expressed in
terms of the original set of data,

\[
\phi_n(x) = \sum_{k=1}^M \theta(x, t_k) A_{nk}
\]  

(9)

we must solve the matrix eigenvalue problem:

\[
\sum_{k=1}^M C_{ij} A_{nk} = \lambda A_{nj}, \text{ for } j = 1, \ldots, M
\]  

(10)

where

\( A_{nj} \): constants associated to the \( n \)th

\( \text{mode; } M: \text{number of snapshots extracted from a transient CFD}

\( \text{simulation; } C: \text{temporal correlation tensor defined by:

\[
C_{ij} = \frac{1}{M} \int_\Omega \theta_i(x, t_k) \theta_j(x, t) dx
\]  

(11)

As \( C \) is symmetric and positive semi-definite, all eigen-
values are real and non-negative and can be ordered as
\( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \). Each eigenvalue \( \lambda_n \), taken individually, re-
resents the energy contribution of the corresponding
eigenfunction. The eigenvectors \( \phi_n \) satisfy the boundary
conditions and can be normalized to form an orthonor-
dmal set. The main property of the POD is its ability to
give the best approximation of the flow in an energetic
sense. The energy contained in the first \( m \) modes is in-
deed always greater than the energy contained in any
other basis, such as the Fourier basis. If POD is applied
to the velocity field, this energy is related to the fluid’s
kinetic energy. In the case of a temperature distribution,
the fact that a mode captures, for instance, 65% of the
energy means in a probabilistic sense that the system
spends 65% of its time executing this mode (Deane and
Sirovich (1991)).

To reach a better accuracy, the method is applied to
the fluctuating field. Then, by keeping only the first \( m \) modes and ignoring the remaining modes, the temperature is written as:

\[
\theta(x, t) \approx \theta_m(x) + \sum_{i=1}^{m} a_i(t) \phi_i(x)
\]  

(12)

where \( \theta_m \) is the mean temperature.

The number of basis functions is chosen according to an energetic criterion which ensures the conservation of maximum energy:

\[
E_c = \sum_{i=1}^{m} \lambda_i / \sum_{i=1}^{n} \lambda_i \geq 90\% \quad \text{and} \quad \lambda_m / \lambda_1 \leq 10^{-2}
\]  

(13)

Only a very small number \( m \) of functions are sufficient to rebuild the temperature distribution. In order to obtain a low dimensional model, equation 12 is substituted into equation 4 and after a change of variable leads to a system of order \( m \):

\[
\begin{aligned}
\dot{a} &= Aa + Bu \\
\dot{\theta} &= Ca + Du
\end{aligned}
\]  

(14)

The low order model 14 gives the behaviour of coefficients \( a_i \) as a function of the input vector. Knowing the basis functions \( \phi_i \), the temperature is then rebuilt with use of the expression 12, leading to the second equation of model 14.

3. CASE STUDY

The aim of this study is to control a \( 4.90 \text{ m} \times 2.82 \text{ m} \times 2.76 \text{ m} \) room equipped with a fan coil having two outlet speeds (1 and 1.5 m/s) and an outlet temperature varying from 16 to 21 \( ^\circ \text{C} \). The fan coil is placed at the right bottom.

In order to use the fixed velocity field, it is necessary to distinguish four different cases: a) jet falling before reaching the ceiling, b) jet sticking to the ceiling and falling rapidly, c) jet sticking to the ceiling and falling before the opposite wall, d) jet flow sticking to the ceiling and reaching the opposite wall). Figure 1 gives an illustration of these 4 cases.

Each one can be defined by a range of outlet temperature and an outlet velocity (Tab. 1).

![Figure 1: Studied cases](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>Outlet temperature ( ^\circ \text{C} )</th>
<th>Outlet velocity ( \text{m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
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</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Parameters of the cases
Case a | 16.0 … 16.7 | 1
Case b | 16.7 … 19.7 | 1
Case c | 19.7 … 21.0 | 1
Case d | 16.0 … 21.0 | 1.5

All walls temperatures are fixed to homogeneous and constant values. The mesh contains 2992 cells. The vector \( \mathbf{u} \) contains wall temperatures and the outlet temperature that is to say the temperature of the point T5 (Fig 2).

In reference to the European standard ISO 7730, the control of temperature is surveyed at three different points in the occupancy zone, respectively at 0.10 m (T1), 1.10 m (T2) and 1.80 m (T3) high. In addition, the temperature at the inlet of the fan coil (T4) is checked. The location of these points is illustrated on Figure 2.

The reduction methodology described in the previous section is applied to each case.

4. RESULTS AND DISCUSSION

In order to evaluate the accuracy of reduced models, we compare their results with CFD simulations and the root mean square error is used:

\[
R_mse = \sqrt{\frac{\sum (x_{ref} - x)^2}{n_x}}
\]  

(15)

where:

- \( x_{ref} \): index stands for reference CFD results
- \( n_x \): number of cells for steady simulations, or number of time step for transient simulations

4.1 High order model

At first, the accuracy of the four high order models has to be checked. It is interesting to have a closer look to case c which is the less advantageous because of the behaviour of the jet. Indeed, the jet is developing quickly along the ceiling without reaching the opposite wall. The accordance with the fixed flow hypothesis is worse than for the other cases. Figure 2 shows the difference of temperature distribution for an outlet temperature of 21 °C and a temperature step of 1°C.

The \( R_mse \) is equal to 0.240 °C for the whole room, 0.205 °C in the occupancy zone and 0.017 °C at the inlet of the fan coil. For all the room, the value is higher because of the difficulty in forecasting the jet flow correctly. The same remark is also valid for the steady state at 20 °C, where the jet flow largely enters in the occupancy zone (Fig 2a) and results in a \( R_mse \) of 0.420 °C in this zone. Nevertheless, the study of the transient period (Fig 3) shows that only points placed in high gradient zone undergo such error (\( R_mse = 0.439 \) °C at T2), whereas points T1 and T3 have \( R_mse \) equal to 0.154 °C and 0.170 °C respectively. The Simulations of the high order model only spend a few minutes whereas CFD simulations need several hours. However this computed time are still too long for real time control. Besides, the order of the model (2992) has to be reduced again to allow the construction of a controller.

4.2 Reduced order model

The POD-based basis functions are extracted from the simulation results of two opposite successive steps of outlet temperature describing all the range of each configuration. For the case (c), 60 snapshots have been extracted from CFD simulations with a time step of 60 s (23 for the step from 19.7 to 21 °C and 37 for the step from 21 to 19.7 °C). They only contain the transient period of the temperature variation. Table 2 gives the number of modes needed to respect the criterion (13).

Table 2: Number of modes for reduction

![Figure 2: Difference of temperature for an outlet temperature of 21 °C between: (a) the high order model and full CFD model, (b) the low order model and full CFD model.](image-url)
Although it is not necessary to keep seven modes for all the cases, all the models are reduced to the same order 7. Comparing the simulation results of the reduced order model with the full CFD model, it results that the reduction does not systematically reduce the accuracy (Fig 2c). In the case of an outlet temperature of 20 °C for instance, the steady state $R_mse$ decreases at 0.317 °C in the working zone. For the steady state at 21 °C, the accuracy of the reduced order model (Fig 2b) is increased at 0.152 °C and 0.124 °C for respectively all the room and the working zone, and it is decreased for the inlet of the fan coil at 0.127 °C, which remains acceptable. Concerning the transient period, results are better than high order model ones, above all for the point T2 (Fig 3b) with a $R_mse$ of 0.229 °C. It is worth remembering that the base of reduction is built without the hypothesis of fixed flow field. Consequently, the base contains information on the velocity field variation, and so enhances the reduced order model compared to the higher order model. For all cases, not only the accuracy is good, but resolution is made in real time. In addition, the size of the reduced order model is only 7, which allows controller design.

5. CONCLUSIONS

The results show a good capacity of reduced models obtained with Proper Orthogonal Decomposition (POD) to predict the temperature in the occupancy zone. The small size of these models allows the controller design for air temperature in indoor spaces. Besides, the use of state space form is suited to the application of modern control theory.

REFERENCES


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