ANALYSIS OF HEAT AND MOISTURE BEHAVIOR IN UNDERGROUND SPACE
BY QUASILINEARIZED METHOD

-ACCURACY RANGE FOR OUTDOOR CLIMATE VARIATION FROM THE REFERENCE-

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ABSTRACT
Coupled heat and moisture transfer equations which describe temperature and moisture fields in the building walls, the earth ground and so on are nonlinear. Due to the nonlinearity, the complicated analysis has to be performed in each time under the different boundary values. If linearization of the system with adequate accuracy can be achieved, a solution can be obtained simply by applying the superposition principle. The purpose of this paper is to clarify the range of outdoor conditions that the quasilinearized method is available on the heat and moisture behavior in the underground space under the natural condition.

INTRODUCTION
Generally, governing equations which describe heat and moisture transfer process in porous materials such as the ground, the building walls and so on are nonlinear. The transfer coefficients (moisture and thermal conductivity) and the moisture capacity strongly depend on the dependent variables such as the moisture content[1,2]. For instance heat and moisture behavior in the underground space are remarkably affected by precipitation, solar radiation and so on[3,4]. These equations are to be solved under varying boundary values, such as the outdoor climate conditions and the room temperature and humidity. This means that a complicated numerical analysis has to be performed in each time when the boundary value used is different. Furthermore, tedious computational efforts in the calculation are needed to obtain the periodic steady state solutions [5] that are useful for the thermal and moisture design of the building envelope. In order to avoid the difficulties and complexities of the nonlinearity of the governing equations, approximation of these equations is needed and is very useful from a view point of the above mentioned design.

The quasilinearized method were presented by Matsumoto[6] as the approximation for the nonlinear heat and moisture transfer equations. These are obtained by expanding the original nonlinear equations and have properties of linearity and time-variant. The quasilinearized method is briefly outlined as follows. When the solution is obtained by solving the nonlinear equations under given boundary values, the variation of the solution for the variation from the given boundary values is calculated by the quasilinearized equations that is linear ones. The approximate solution for another boundary values is calculated by superposing the linear solution on the nonlinear solution. Matsumoto and Nagai [7] and Matsumoto and Tanaka[8] discussed the application of moisture variation on the building walls and confirmed the usefulness of the quasilinearized method. The analytical model on heat and moisture transfer in the underground space have been presented [4]. In the previous paper[9] we clarified the range of the room and the outdoor temperature that the quasilinearized method is available on the heat load of the underground space. As the results, the allowable range of approximasion is \( \pm 10[{\degree C}] \) in the variation of the annual average outdoor temperature and \( \pm 15[{\degree C}] \) in the variation of the annual average indoor temperature. Also it was shown that the step responses of heat load of the underground space in the finite variation of the outdoor and room temperature are available to be treated as the time invariant. The purpose of this paper is to clarify the range of outdoor conditions that the quasilinearized method is available on the heat and moisture behavior in the underground space under the natural condition.

FUNDAMENTAL EQUATIONS AND QUASILINEARIZED METHOD

FUNDAMENTAL EQUATIONS
Coupled heat and moisture transfer model is derived with the assumption of local equilibrium between the liquid phase and the gas phase of the water without solid phase (ice) [1,2]. Heat and moisture conservation equations are as follows.

\[
\begin{align*}

\frac{\partial T}{\partial t} & = \nabla \cdot \left( \rho \cdot \lambda \cdot \nabla T \right) + q
\end{align*}
\]

where

\[
\begin{align*}

k(\mu, T) \frac{\partial \mu}{\partial t} & = \nabla \cdot \left( \nabla \mu \left( \nabla \mu - n g \right) + \nabla \cdot \lambda \nabla T 
\right)
\end{align*}
\]

Boundary conditions of heat and moisture transfer at

\[
- \lambda_{(\mu)} \left( \frac{\partial T}{\partial n} - n g \right) - \lambda_{(\mu)} \frac{\partial T}{\partial n} = \alpha_{(\mu)} (\mu - \mu) \cdot (\mu - \mu) + \alpha_{(\mu)} (\mu - \mu) + q,
\]
The variation of the solution for the variation from the reference boundary values is normally obtained by solving the quasilinearized equations. The other way is that the linear solution is obtained from the difference between two nonlinear solutions under the given different boundary values. In the previous paper[4], we used the quasilinearized equations to obtain the linear solution for the variation from the reference boundary value. In this paper the variation of the solution is obtained as the difference between two nonlinear solutions for the given different boundary values respectively. There are two reasons not to use the quasilinearized equations. One is that it takes too much nodes and time to calculate the quasilinearized equations. Another is that the high stability and accuracy seems to be obtained in this case since Tanaka et al indicated the above method on the building wall surface. These parameters are particularly dependent on the former. The nonlinearity of the governing equations is due to the dependency of these coefficients on the state variables. When the air in the underground space is treated as 1 material point, equations of heat and moisture balance of the air are

\[ c v \frac{\partial T}{\partial t} = \sum S \cdot \alpha(T_i-T) + c v N V (T_s-T_s) + q_s \cdots \cdots (9) \]

where subscript i denotes the node point on the wall surface and M denotes the total number of it. These equations are linear equations when \( N_s \), \( q_s \), and \( J_s \) keep constant values. Dependent variables in these equations are temperature and water vapor partial pressure. The relation between water chemical potential \( \mu \) and water vapor partial pressure \( p \) is as follows.

\[ \mu = R \ln \left( \frac{p}{p_{sat}} \right) \cdots \cdots (11) \]

After all, dependent variables are absolute temperature \( T \) and water chemical potential \( \mu \).

**QUASILINEARIZED METHOD**

The concept of quasilinearized method is that the approximate solution is obtained by superposing the linear solution on the nonlinear solution. The nonlinear solution is obtained by solving the nonlinear equations under the reference boundary value. The linear solution is the variation of the solution for the variation from the boundary values hence the solution means the sensitivity for the variation of the boundary value. Once the linear solution and the nonlinear solution are obtained, the solution for an arbitrary boundary value is calculated by using the principle of the superposition. The quasilinearized method is useful if the range of the variation that the method is available is large. The variation of the solution for the boundary values is solved using the quasilinearized equations[6]. The way of calculating the linear solution for the variation from the reference boundary values is normally obtained by solving the quasilinearized equations. The other way is that the linear solution is obtained from the difference between two nonlinear solutions under the given different boundary values. In the previous paper[4], we used the quasilinearized equations to obtain the linear solution for the variation from the reference boundary value. In this paper the variation of the solution is obtained as the difference between two nonlinear solutions for the given different boundary values respectively. There are two reasons not to use the quasilinearized equations. One is that it takes too much nodes and time to calculate the quasilinearized equations. Another is that the high stability and accuracy seems to be obtained in this case since Tanaka et al indicated the above method on the building wall surface. These parameters are particularly dependent on the state variables. When the air in the underground space is treated as 1 material point, equations of heat and moisture balance of the air are

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**NUMERICAL ANALYSIS**

**PROPERTIES OF MATERIALS AND TRANSFER COEFFICIENTS**

A soil of the earth ground is the Plainfield Sand. The thermal and moisture properties measured by Jury[10] are used in this analysis. The structure of the wall of the underground space is consist of the concrete. The properties of concrete are composed of the value based on the measured data[11,12,13]. Heat and moisture transfer coefficients and other properties are shown in Table 1.

**BOUNDARY VALUES FOR THE REFERENCE SOLUTION**

Boundary conditions at surface contacted with air are third kind. Vertical boundary of the earth ground far from the underground space is treated as surfaces of no flow of heat and moisture, the lower boundary condition is first kind. The boundary values are outdoor climate, room climate, lower boundary conditions in the earth ground. Outdoor climate as which outdoor temperature, relative humidity, solar radiation and precipitation are used, are measured from 1991 to 1992, at Kobe in Japan. These are shown in Figures 1-4. Also longwave radiation is calculated by Brunt’s formula. The outdoor temperature and the relative humidity are developed to Fourier series. The annual average of outdoor temperature and relative humidity are 16.0 [°C] and 61 [%], respectively. Annual solar radiation and precipitation are 4567[MJ] and 1211 [mm], respectively. Room air is ventilated in constant rate and there is no heat and moisture sources. The lower boundary of the earth ground is 14.4 [m] in depth. The lower boundary temperature and water chemical potential is kept at 16.4 [°C] and -1 [J/Kg], respectively.
Table 1: Heat and moisture transfer coefficients and other properties

<table>
<thead>
<tr>
<th>Item</th>
<th>Value [unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat capacity of air</td>
<td>1200 [J/m³ K]</td>
</tr>
<tr>
<td>Moisture capacity of air</td>
<td>$7.5 \times 10^6$ [kg/m³ Pa]</td>
</tr>
<tr>
<td>Heat transfer coefficient</td>
<td>Outdoor air layer</td>
</tr>
<tr>
<td></td>
<td>Indoor air layer</td>
</tr>
<tr>
<td>Moisture transfer coefficient</td>
<td>Outdoor air layer</td>
</tr>
<tr>
<td></td>
<td>Indoor air layer</td>
</tr>
<tr>
<td>Solar adsorption ratio</td>
<td>0.8 [-]</td>
</tr>
</tbody>
</table>

Table 2: Combinations of variations of outdoor boundary values

<table>
<thead>
<tr>
<th>Variational function of boundary value</th>
<th>Air temperature [°C]</th>
<th>Relative humidity [%]</th>
<th>Solar radiation [W/m²]</th>
<th>Precipitation rate [mm/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>$0.05 \times q_s(t)$</td>
<td>$0.05 \times J_s(t)$</td>
<td></td>
</tr>
</tbody>
</table>

*) $q_s(t)$ and $J_s(t)$ are the reference values

VARIATIONS OF BOUNDARY VALUES FROM THE REFERENCE

The variational functions of the boundary value to obtain the linearized solution are shown in Table 2 as the difference between the reference boundary value and the specified boundary value. With respect to outdoor temperature and humidity, the variation of the boundary value use Heaviside function, hence the solution is a step response. In other ones the variation is the reference value multiplied by a constant value.

PROBLEMS CALCULATED

The subject for analysis is the basement shown in Figure 5. Indoor air is in natural condition without heat and moisture sources and with a constant air change rate, 0.5 [1/h]. Numerical analysis is performed by explicit finite difference method. The number of nodes is 951 and the minimum discrete size of space is 0.02 [m] at the inside surface of the wall. The discrete size of time is 30 [s]. All the solutions are confirmed to become in periodic steady state.
RESULTS AND DISCUSSIONS

THE CASE OF THE VARIATIONS OF THE OUTDOOR TEMPERATURE

Figures 6, 7 show the annual variation of room temperature and relative humidity in the underground space in the case of $\Delta T_o = +10{^\circ\text{C}}$, respectively. The solid line in the figure shows the exact solution calculated by nonlinear equations. The dotted line shows the approximate solution calculated by quasilinearized method. The chain line shows the solution calculated by nonlinear equations under the reference boundary value (called the reference solution). As the annual average outdoor temperature increases, room temperature increase largely but relative humidity hardly changes. The variation of the solution for the variation of the outdoor temperature is almost the constant through the year. The approximate solutions on room temperature and relative humidity agree well with the exact solutions.

Figure 8, 9 shows the distribution of heat flux and moisture flux on the envelope wall of the underground space in the case of $\Delta T_o = +10{^\circ\text{C}}$, respectively. The approximate solutions with the water flux agree well with the exact solutions. In the case of $\Delta T_o = -10{^\circ\text{C}}$, the accuracy of the approximate solutions on room temperature and humidity is the same as the case of $\Delta T_o = +10{^\circ\text{C}}$. Hence the allowable range of the approximation on room temperature and humidity of the underground space is $\pm 10{^\circ\text{C}}$ in the variation of the outdoor temperature.

THE CASE OF THE VARIATION OF THE OUTDOOR RELATIVE HUMIDITY

Figures 10, 11 show the annual variation of room temperature and relative humidity in the underground space in the case of $\Delta \mu_o = +18000$ [J/kg][relative humidity =+10[%]], respectively. As the annual average outdoor relative humidity increases, the room tempera-
The accuracy of approximation on room temperature and relative humidity agrees well with the exact solution except for in summer. But the approximate solution in summer is higher than the exact solution and is riskier value from the point of view of the avoidance of high humidity, hence this value is allowable. In the case of $\Delta \mu_r = -18000 \text{[J/kg]}$ (relative humidity $= -10\%$), the accuracy of the approximate solutions is the same as the case of $\Delta \mu_r = +18000 \text{[J/kg]}$ in room temperature and relative humidity. Therefore the allowable range of the approximation on room temperature and relative humidity in the underground space is $\pm 10 \%$ in the variation of the outdoor relative humidity.

THE CASE OF THE VARIATION OF THE SOLAR RADIATION

Figures 12, 13 show the annual variation of room temperature and relative humidity in the underground space in the case of $\Delta q_s = +0.4 \times q_s$, the solar radiation of 1.4 times as much as the reference value, respectively. As the solar radiation increases, the room temperature increases and the room relative humidity decreases. The annual average temperature increases $1.0 \text{[°C]}$ from the reference solution. The approximate solutions in the room temperature and relative humidity agree well with the exact solutions. In the case of $\Delta q_s = -0.4 \times q_s$, the accuracy of the approximate solutions is the same in room temperature and relative humidity, the allowable range of the approximation on room temperature and relative humidity in the underground space is 0.6 -1.4 times as much as the reference value of the solar radiation.

THE CASE OF THE VARIATION OF THE PRECIPITATION

Figures 14-17 show the annual variation of room temperature and relative humidity in the underground space in the case of $\Delta J_r = \pm 0.5 \times J_r$, the precipitation of 0.5 and 1.5 times as much as the reference value, respectively. As the precipitation increases(decreases), the room temperature slightly decreases(increases) and the room relative humidity increases(decreases). The annual average room temperature in the precipitation of 0.5 and 1.5 times as much as the reference value increases $-0.3$ and $+0.6 \text{[°C]}$, respectively. The approximate solutions on room temperature and relative humidity agree with the exact solutions except for the relative humidity in the case of $\Delta J_r = -0.5 \times J_r$. The allowable range of the approximation on room climate of the underground space is 0.5-1.5 times as much as the reference value of the precipitation if the error of relative humidity in the case of $\Delta J_r = -0.5 \times J_r$ is allowable. In this case, it seems that the variation of the room temperature is small against the variation of the precipitation. To clarify the detailed
sensitivity of the precipitation against the room temperature, the exact solution under 0-2.0 times as much as the reference are investigated. Figure 18 shows the annual average room temperature with the several rate against the reference precipitation. The annual average temperature remarkably changes in the low rate of the reference and slightly changes in the high rate. Therefore, the precipitation remarkably affects the room temperature in the underground space. The sensitivity of the precipitation is high in a small amount of the precipitation and is low in about the reference.

CONCLUSIONS

In this paper, the range of outdoor conditions that the quasilinearized method is available on the heat and moisture behavior in the underground space under the natural condition are clarified. The outdoor conditions investigated are outdoor temperature, relative humidity, solar radiation and precipitation. The variations of the boundary value are that of the annual average value on outdoor temperature and relative humidity and are that of the constant ratio of the reference value in each time on solar radiation and precipitation. The results are shown as follows.

1) The allowable range of the approximation on room temperature and humidity of the underground space is ±10°C in the variation of the outdoor temperature.
2) The allowable range of the approximation on room climate of the underground space is ±10 [%] in the variation of the outdoor relative humidity.
3) The allowable range of the approximation on room climate of the underground space is 0.6-1.4 times as much as the reference value of the solar radiation.
4) The allowable range of the approximation on room climate of the underground space is 0.5-1.5 times as much as the reference value of the precipitation.

The accuracy of the superposition of the variations of the solution under the several outdoor conditions need to be discussed in the future. In Japan, the variation of outdoor temperature, relative humidity, solar radiation and precipitation are ranging 5.7-22.4°C in the annual average, 64-80[%] in the annual average, 4000-5800 [MJ] in the annual amount and 810-3000[mm] in the annual.
REFERENCES


Yokendo (1990)


Nomenclature

c = apparent heat capacity of the material [J/m³ K]
c_j = specific heat of liquid water [J/kg K]
c_q = heat capacity of moist air [J/kg K]
c_p = water vapor capacity of moist air [kg/m³ Pa]
g = external mass force (gravity) [N/kg]
J = moisture diffusion [kg/m² s]
J_i = moisture diffusion in liquid phase [kg/m² s]
J_r = surface liquid water production (precipitation) [kg/m² s]
J_s = room air moisture production [kg/m³ s]
k = water capacity of porous material in terms with water chemical potential [kg/m³ J/kg]
n = inward normal vector to the material []
N_i = air change rate of room air [times/hour]
p = water vapor partial pressure [Pa]
p_a = water vapor partial pressure of outdoor air [Pa]
p_r = water vapor partial pressure of room air [Pa]
p_{sat} = saturated water vapor partial pressure [Pa]
q_a = room air heat production [W/m²]
q_s = heat production in terms of radiation on the surface [W/m²]
R_v = universal gas constant for water vapor [Pa m³/kg K]
S_i = area of i th part of the wall [m²]
T = temperature [K]
T_r = outdoor air temperature [K]
T_p = temperature of precipitation [K]
T_s = temperature on the surface [K]
t = time [s]
V = volume of the room [m³]
α = heat transfer coefficient [W/m² K]
α_e = α + rα_p = effective heat transfer coefficient [W/m² K]
α_p = rα_p = heat transfer coefficient related to water chemical potential [W/(m² s J/kg)]
α_p = vapor transfer coefficient related to water chemical potential difference [kg/(m² s J/kg)]
α_p = vapor transfer coefficient related to temperature difference [kg/m² K]
λ = thermal conductivity [W/m K]
λ_c = λ + rλ_p = effective thermal conductivity [W/m K]
λ_p = rλ_p = thermal diffusivity related to water chemical potential [W/(m s J/kg)]
λ_c = λ_p + λ_m = moisture conductivity related to water chemical potential gradient [kg/m s J/kg]
λ_p = moisture conductivity in gas phase
related to water chemical potential gradient

\[ \dot{\lambda}_{\text{mol}} = \text{moisture conductivity in liquid phase related to water chemical potential gradient} \quad [\text{kg/m s J/kg}] \]

\[ \dot{\lambda}' = \dot{\lambda}'_{g} + \dot{\lambda}'_{l} = \text{moisture conductivity related to temperature gradient} \quad [\text{kg/m s K}] \]

\[ \dot{\lambda}'_{g} = \text{moisture conductivity in gas phase related to temperature gradient} \quad [\text{kg/m s K}] \]

\[ \dot{\lambda}'_{l} = \text{moisture conductivity in liquid phase related to temperature gradient} \quad [\text{kg/m s K}] \]

\[ \rho_w = \text{density of liquid water} \quad [\text{kg/m}^3] \]

\[ \mu = \text{water chemical potential referred to free water} \quad (= RvT ln (p/p_{sat})) \quad [\text{J/kg}] \]

\[ \psi = \text{volumetric moisture content} \quad [\text{m}^3/\text{m}^3] \]

**Subscripts**

- \( b \) = boundary value
- \( g \) = moisture in gas phase
- \( i \) = \( i \)th node point
- \( l \) = moisture in liquid phase
- \( o \) = outdoor air
- \( p \) = precipitation
- \( r \) = room air
- \( s \) = surface
- \( T \) = temperature
- \( \mu \) = water chemical potential
- \( 1,2 \) = material number