

# APPLICATION OF MULTI-OBJECTIVE GENETIC ALGORITHMS AND $CRI_{(C)}$ & $CRI_{(H)}$ ANALYSIS TO INDOOR HUMIDITY ENVIRONMENT DESIGN OPTIMIZATION

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## ABSTRACT

This study presents a multi-objective optimum design method for reliable indoor humidity environments based on the appropriate use of moisture-conditioning materials. In this paper, (1) a transient prediction of indoor air temperature and humidity in a model living room is developed by employing indices for the contribution ratio of indoor climate ( $CRI_{(C)}$ ) and contribution ratio of indoor humidity ( $CRI_{(H)}$ ), and (2) an optimal design method is developed using multi-objective genetic algorithms (MOGA) and the transient prediction above. In order to confirm the validity of the proposed optimum design system, a conventional model living room with a number of moisture sources is set up as an analytical target and the optimum amount and arrangement of moisture-conditioning materials as design parameters are analyzed to achieve a reliable environmental design for indoor humidity. As a result, optimal solutions are obtained by means of MOGA and the effectiveness of this approach is demonstrated for designers.

## KEYWORDS

Indoor humidity environment, Moisture-conditioning material,  $CRI_{(C)}$  and  $CRI_{(H)}$  analysis, MOGA

## INTRODUCTION

In recent years, environmental problems causing worldwide damage have become a serious concern for mankind, and efforts to combat and resolve these problems are being undertaken on a global scale. In order to save energy, buildings are usually being built to be more airtight and insular. In these highly airtight buildings, increasing amounts of steam vapor from numerous moisture sources and volatile organic compounds (VOCs) from building materials and furniture are accumulating, which could induce condensation within the building envelope components and affect the occupants' health. Under these circumstances, air-conditioning systems are introduced to control the indoor humidity. Although great efforts have been made to reduce the risk of condensation; the indoor environment is still highly susceptible to raised levels of humidity such as from the careless opening of windows by occupants, sometimes resulting in a large degree of momentary moisture generation. Thus, it is necessary to efficiently control the indoor humidity and create a better residential environment. On the other hand, moisture-conditioning materials are widely used in buildings as a result of their excellent ability to control humidity. But the amount of such material required and its location would benefit from further discussion in order to ensure the optimal utilization.

Recently, much work has been done to assist designers in indoor thermal environmental design. Kim et al. (2000) developed a two-step optimal design method using coupled simulation of computational fluid dynamics (CFD) and genetic algorithms (GA) to optimize the location of both the air-conditioning supply inlet and the radiative cooling panels. Optimization of hybrid air-conditioning systems that use natural ventilation by GA and CFD has been conducted by Lee et al. (2004) in order to enhance the

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energy-saving effect. However, further environmental improvements are required to fine-tune indoor humidity, since there is almost no precedent focusing on the optimization of indoor humidity.

On the other hand, in order to optimize the utilization of moisture-conditioning materials, first of all, a precise long-range prediction is necessary in which the effect of the materials' buffer is taken into account. A uniform simulation model that considered the adsorption and desorption phenomenon of interior materials has been widely used (Diasty 1992). In this model, the temperature and humidity of the whole room space were considered to be uniform (complete diffusion). However, it is difficult to evaluate the influence of vapor distribution near the materials and the contributions of those materials at different positions within a room space. Recently, coupled simulation models using CFD and vapor diffusion through building materials have been proposed by Tsay et al. (2006). However, the coupled simulation turned out to be time-consuming and difficult to handle for long-range predictions. A simplified coupled simulation model based on CFD has also been proposed in order to perform a transient prediction of indoor humidity within a room in which silica gel is arranged on the floor as a moisture-conditioning material (Hu 2006). In that model, since the silica gel is a porous material with negligible resistance to interior vapor diffusion, the diffusion coefficient of the vapor in this material is assumed to be infinity and the material can reach a state of adsorption (desorption) equilibrium simultaneously in the adsorption and desorption processes. Although the simplified coupled simulation model with CFD significantly shortened the analysis time, it is not suitable for application to this optimization investigation since it is still hard to handle for annual-scaled simulations.

Moreover, in order to understand the mechanism of air temperature composition in a room, Kato et al. (1994) defined three new scales as the contribution ratios for indoor climates ( $CRI_{(C)}$ ) in order to assess the contribution of each heat source to air temperature distribution. Meanwhile, Hu et al. (2007) investigated contributions to indoor humidity from numerous moisture sources and defined the new index as the contribution ratios for indoor humidity ( $CRI_{(H)}$ ) in order to offer an effective control and design methodology for indoor air quality (IAQ). Moreover, as follow-up research (Hu 2007), the  $CRI_{(C)}$  and  $CRI_{(H)}$  indices are employed to perform a long-range coupled prediction in a model living room in which moisture-conditioning material is arranged. Since the  $CRI_{(C)}$  and  $CRI_{(H)}$  analysis method proved to be not only precise but also applicable, in this study, the optimum inquiry is based on long-range prediction by means of  $CRI_{(C)}$  and  $CRI_{(H)}$  analysis.

This study aims to achieve a reliable indoor humidity environment by optimizing the utilization of the moisture-conditioning materials. In specific terms, it presents a multi-objective optimum design program coupled with a long-range transient simulation by means of  $CRI_{(C)}$  and  $CRI_{(H)}$  analysis, which will be introduced in the following section.

## COUPLED SIMULATION BY MEANS OF $CRI_{(C)}$ AND $CRI_{(H)}$

### Definitions of $CRI_{(C)}$ and $CRI_{(H)}$

$CRI_{(C)}$  indicates the ratio of temperature rise at a point from one individual heat source to the absolute value of the temperature rise from the heat source with uniform distribution of the same amount of heat (Kato, 1994). This index clearly indicates how far the generated heat diffuses within the space. In other words, it indicates the range and the degree of influence of each heat source and thus this value doesn't mean its absolute intensity (i.e. it is normalized by its own value of perfect mixing condition). The definition of  $CRI_{(C)}$  is shown in Equation (1).

$$CRI_{(C)}(x, k) = \frac{\delta\theta(x, k)}{\theta_k} \quad \theta_k = \frac{\Theta_k}{C_p \cdot \rho \cdot Q} \quad (1)$$

$\delta\theta(x, k)$ : temperature rise (fall) based on the standard status at a point x based on the  $k^{\text{th}}$  heat source [ $^{\circ}\text{C}$ ]

$\theta_k$ : temperature rise (fall) under perfect mixing conditions based on the  $k^{\text{th}}$  heat source [ $^{\circ}\text{C}$ ]

$\Theta_k$ : heat flux generated from the  $k^{\text{th}}$  heat source [J/s]

In contrast to  $CRI_{(C)}$ ,  $CRI_{(H)}$  indicates the ratio of humidity rise (or fall) at a point from an individual moisture source to the humidity rise (or fall) under perfect mixing conditions for the same moisture source. This index clearly explains how far the moisture generated from a notable moisture source diffuses throughout the space (Hu, 2007). Equation (2) shows the definition of the index.

$$CRI_{(H)}(x, n) = \frac{\delta X(x, n)}{X_n} \quad X_n = \frac{q_n}{\rho \cdot Q} \quad (2)$$

$\delta X(x, n)$ : humidity rise (fall) based on the standard status at a point x based on the  $n^{\text{th}}$  moisture source [kg/kg']

$X_n$ : humidity rise (fall) under perfect mixing conditions based on the  $n^{\text{th}}$  moisture source [kg/kg']

$q_n$ : moisture flux generated from the  $n^{\text{th}}$  moisture source [kg/s]

$CRI_{(C)}$  and  $CRI_{(H)}$  are indices which explain the contributions to indoor thermal and humidity environment from all kinds of heat and humidity sources, both being obtainable by performing a steady-state CFD simulation (Hu, 2007).

### $CRI_{(C)}$ and $CRI_{(H)}$ analysis

Figure 1 shows the composition of the indoor climate, which is comprised of numerous heat and moisture sources and ventilation conditions (In this research, the moisture-conditioning material is treated as both a heat and moisture source with latent heat generated in the adsorption and desorption processes also being taken into account). The distribution of air temperature (humidity) is greatly

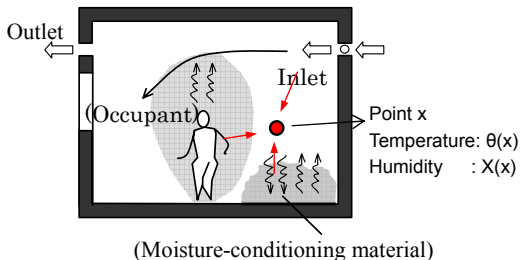


Figure 1. An image of the linear synthesis of indoor temperature and humidity

affected by airflow and the positions of heat (moisture) sources. Since both the indoor temperature and humidity field can be assumed to have a linear structure if the indoor flow field is dominated by forced convection rather than natural convection, if the air temperature (humidity) distribution caused by each heat (moisture) source is analyzed respectively, the indoor air temperature (humidity) can be obtained as a linear summation of the air temperature (humidity) rise (or fall) caused by each heat (moisture) source based on the standard status (Equations (3) and (4)). Consequently, the linear synthetic air temperature (humidity) is almost identical to the one in the original air temperature (humidity) field.

$$\theta(x) = \sum_{k=1}^K \{CRI_{(C)}(x, k) \times \theta_k\} \quad (3) \quad X(x) = \sum_{n=1}^N \{CRI_{(H)}(x, n) \times X_n\} \quad (4)$$

$\theta(x)$ : linear synthetic air temperature rise based on contributions from all heat sources [°C]

$X(x)$ : linear synthetic air humidity rise based on contributions from all moisture sources [kg/kg']

$CRI_{(C)}(x, k)$ : contribution ratio of indoor temperature corresponding to the  $k^{\text{th}}$  heat source at point x [-]

$CRI_{(H)}(x, n)$ : contribution ratio of indoor humidity corresponding to the  $n^{\text{th}}$  moisture source at point x [-]

Since the moisture-conditioning material is treated as both a heat and moisture source, heat flux ( $\Theta_s$ ) and moisture flux ( $q_s$ ) generated from the material are treated as the boundary conditions for the  $CRI_{(C)}$  and  $CRI_{(H)}$  analysis.

$$m \cdot c_p \cdot \frac{d\theta}{dt} = \alpha' (X(D) - X_S) \cdot L \cdot A + \alpha \cdot (\theta(D) - \theta_S) \cdot A \quad (5)$$

$$m \cdot \frac{dM}{dt} = \alpha' (X(D) - X_S) \cdot A \quad (6)$$

$\theta$ : temperature of material [°C]

M: water content of material [kg'/kg]

$\theta(D)$  and  $X(D)$  represent the temperature and humidity of air adjacent to the material. Since both could be synthesized linearly (Equations (3) and (4)), the material's temperature and water content could be calculated by the laws of conservation of energy and mass (Equations (5) and (6)); again, the obtained heat flux ( $Q_s$ ) and moisture flux ( $q_s$ ) from the material are fed back as boundary values for the  $CRI_{(C)}$  and  $CRI_{(H)}$  analysis (Equations (3) and (4)) and consequently lead to new air temperature and humidity. The feedback loop is repeated in single time steps until reasonable convergence is achieved for both air and material temperature and humidity. In this study, a fully implicit method is employed to discretize the time scheme with a calculation interval of 10 minutes. By employing the coupled analysis above, a precise long-range prediction could be performed (Hu 2007).

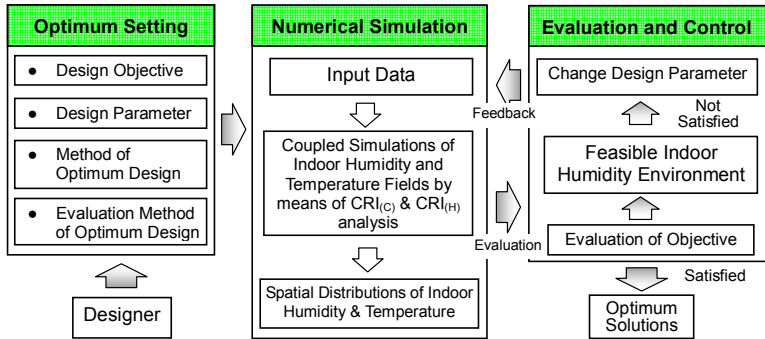


Figure 2. The structure of the optimum design system

## STRUCTURE OF THE OPTIMUM DESIGN SYSTEM

Figure 2 shows a flowchart of the optimum design system for indoor humidity environmental design based on GA and coupled simulation of  $CRI_{(C)}$  and  $CRI_{(H)}$ . The system is composed of three elements: (1) optimum settings by designers, (2) numerical simulations of the indoor environment by means of coupled simulation of  $CRI_{(C)}$  and  $CRI_{(H)}$ , and (3) GA-based evaluation of the optimum solution candidate and control of the GA inquiry process. In the first part, the design objective, parameters, method of optimum design and evaluation are specified by the designers. Then, in the second stage, the  $CRI_{(C)}$  and  $CRI_{(H)}$  indices are employed to perform a long-range coupled simulation in a model living room in which moisture-conditioning material is arranged, and meanwhile, spatial distributions of indoor humidity and temperature are also obtained as a result of the prediction above. The third part is called evaluation and control, which judges whether the optimum evaluation value of the optimum solution candidate calculated in the second part is suitable or not. If the design objective is not satisfied, the combination of design parameters is changed and fed back to the second part. By repeating the feedback loop, the optimum solutions are finally obtained once the design objective is satisfied.

## MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

Most realistic optimization problems, particularly those in engineering field designs, require the simultaneous optimization of more than one objective function. Additionally, in most situations where values interact, it is almost impossible to get the absolute optimum solutions, so we must adopt various trade-offs or compromises. When we have a set of solutions such that we can't improve any objective further without at the same time worsening another, then we have what is called the 'Pareto-optimal set' or 'Pareto front'. In such a case, all the other lesser solutions are said to be 'dominated' by these better ones and can be discarded. The set of these 'can't do better' trade-offs (the non-dominated set) contains all the acceptable solutions, different combinations of the objectives and we then need to

select one or more of these for practical use by more intuitive means – their fitnesses being exactly the same, i.e. the 'Decision Maker' (DM) needs to prioritize the criteria or establish preferences (incorporate subjective values) appropriate to the full contextual situation.

The problems' characteristics stated above make multi-objective genetic algorithms (MOGA) an appropriate tool to be employed in this study. In order to confirm the validity of the proposed optimum design system (Figure 2), a tradition living room with moisture-conditioning material arranged is set up as the analytical target in this research (Figure 3).

### Design objectives

This research aims to assist designers in achieving a reliable environmental design for indoor humidity by optimizing the utilization of moisture-conditioning material (silica gel) during the mid-term season (April ~ July). In specific terms, it prevents the indoor environment from often being subject to high humidity thanks to the appropriate use of the material, thus reducing the risk of condensation within building envelope components and offering better IAQ. Meanwhile, the initial cost of the material is also selected as one of the objectives in order to consider the economic aspects.

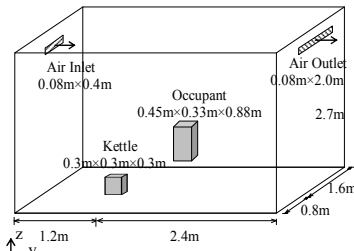


Figure 3. Schematic view of the model living room

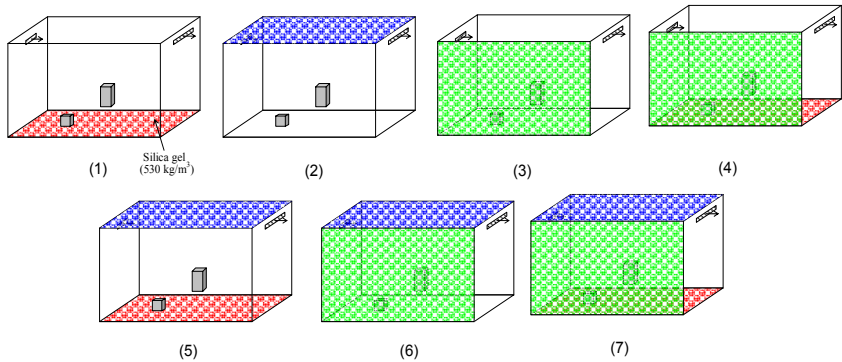


Figure 4. Schematic view of different combinations of materials

### Analytical target and variables

Figure 3 shows a schematic view of a traditional living room to be analyzed. It is assumed that a natural ventilation system is adopted in the mid-term season (April ~ July) so as to save energy. The ventilation rate is set up as 6 (1/h). In the room, an occupant is treated as a constant heat (54 W) and moisture source (80 g/h). Use of a kettle is also considered and a schedule of kettle use (moisture generation profile) is assumed according to the occupants' lifestyles: 1 hour from 7 a.m. to 8 a.m. in the morning, 2 hours from 12 a.m. to 2 p.m. at midday and 2 hours from 6 p.m. to 8 p.m. in the evening, with the moisture load of the kettle being 540 g/h. Moreover, this study is carried out in the mid-term season from April to the middle of July based on AMeDAS weather data for Tokyo.

Two types of design variables are set up in this study. Figure 4 shows the different places in which moisture-conditioning material is arranged and their combinations, which are selected as the design variables (discrete). The thickness of the moisture-conditioning material arranged on each place is also set up as a continuous design variable which varies from 0.01 m ~ 0.5 m, respectively.

### Objective functions

The aims of this study are (1) to prevent the indoor environment from often being subject to high humidity, and thus create a reliable indoor humidity environment, and (2) to consider the economic aspects at the same time.

(1) The optimization evaluation method is described as follows. The cumulative ratio of relative indoor humidity is considered to be an important index for evaluating indoor humidity environment (Hu et al. 2006). Yanagi and Ikeda (2005) investigated microbial contamination within the air-conditioning systems of eight buildings in Tokyo. In their investigations, microbial contamination was detected in almost all buildings in which the cumulative ratio of relative indoor humidity exceeding 70% is more than 30%. Meanwhile, a relative humidity from 70% to 95% is considered conducive to microbial growth. In this study, the entire indoor space is taken into consideration and the cumulative ratio of average relative indoor humidity exceeding 80% is set up as the objective function, which can be expressed as:

$$CR = \frac{T_A}{T_{total}} \quad (7)$$

CR: Cumulative ratio of average relative indoor humidity exceeding 80% [%]

$T_A$ : Number of hours when relative indoor humidity exceeded 80% [hr]

$T_{total}$ : Total analysis time [hr]

In this study, the individual with the minimal objective function is considered to be the best solution.

(2) Average relative indoor humidity within this analysis.

$$\bar{X} = \frac{1}{T_{total}} \sum_{i=1}^{T_{total}} X_i \quad (8)$$

$\bar{X}$ : Average relative indoor humidity during this analysis [kg/kg]

$X_i$ : Average relative indoor humidity in  $i^{\text{th}}$  time steps [kg/kg]

$T_{total}$ : Total analysis time [hr]

In this study, the individual with the minimal objective function is considered to be the best solution.

(3) Since excessive fluctuation of indoor relative humidity could lead to deterioration in the building envelope component, here, standard deviation of the average relative indoor humidity within the entire analytical period is taken into account and the individual with the minimal objective function is considered to be the best solution. The equation of standard deviation is expressed as follows:

$$\sigma = \sqrt{\frac{1}{T_{total}} \sum_{i=1}^{T_{total}} (X_i - \bar{X})^2} \quad (9)$$

$\sigma$ : Standard deviation of average relative indoor humidity [-]

(4) The three objectives stated above are general concerns about the indoor humidity environment, however, in reality, people always consider economic aspects simultaneously. Here, the initial cost of the material and the construction fee should also be considered, which are formulated as:

$$F_{BC} = \sum \{ \rho \cdot d \cdot s \cdot P + Cons \} \quad (10)$$

$F_{BC}$ : Economic function related to initial cost [Yen]

$\rho$   $d$   $s$ : Mass of material used [kg]

$P$ : Market price of material [Yen/kg]

Cons: Construction costs for moisture-conditioning material [Yen]

In this study, the individual with the minimal objective function is considered to be the best solution.

## RESULTS AND DISCUSSIONS

Figure 5 indicates a series of trade-off relationships between different objective functions. The

relationship between standard deviation of the average relative indoor humidity and initial cost is shown in Figure 5(a), while Figure 5(b) presents the relationship between average relative indoor humidity and standard deviation. Figure 5(c) indicates the relationship between standard deviation and the cumulative ratio of average relative humidity exceeding 80%. It can be seen that the randomly produced individuals are widely distributed due to the properties of MOGA, while the final individuals are clustered in the lower left corner. Since it is hard to obtain an actual global Pareto front for most practical problems, the individuals in the areas enclosed can be regarded as the Pareto front. The reason for forming a Pareto front in Figure 5(a) is that if more material is arranged (which results in higher initial cost), the capacity of the moisture buffer is much bigger and therefore, the standard deviation of the average relative indoor humidity decreases since the excessive fluctuation of relative indoor humidity is much more alleviated. In Figure 5(b) below, however, if a huge amount of material is arranged (which results in lower standard deviation of the average relative indoor humidity), the indoor environments will be subject to high humidity due to the desorption process after the material first absorbs any excessive moisture vapor from within the air. In Figure 5(c), some of the solutions are clustered in the lower left corner showing the trade-off relationship between two objective functions while the trade-off relationship between the others are not so clear-cut since they are clustered in the upper areas.

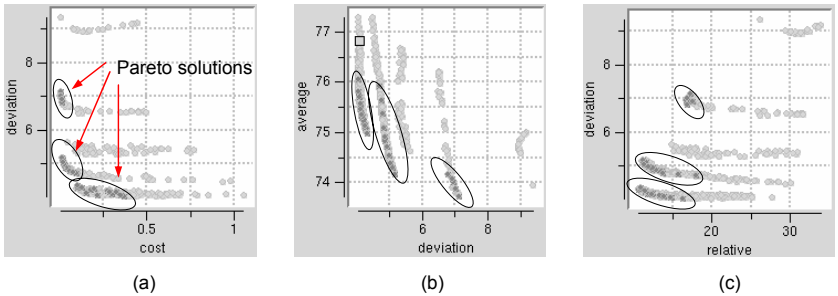


Figure 5. Relationships between different objective functions

No.	No. 1	No. 2	...	No. 29	No. 30
Pareto solutions			...		
Combination	7	7	...	2	2
Design variables (Thickness)	$d_{\text{floor}} = 0.027 \text{ m}$ $d_{\text{ceiling}} = 0.047 \text{ m}$ $d_{\text{sidewall}} = 0.301 \text{ m}$	$d_{\text{floor}} = 0.035 \text{ m}$ $d_{\text{ceiling}} = 0.020 \text{ m}$ $d_{\text{sidewall}} = 0.299 \text{ m}$	...	$d_{\text{ceiling}} = 0.014 \text{ m}$	$d_{\text{ceiling}} = 0.010 \text{ m}$
Objective functions	$\sigma = 4.04$ CR = 16.21% X = 76.06% $F_{\text{BC}} = 0.3754$	$\sigma = 4.06$ CR = 14.75% X = 75.82% $F_{\text{BC}} = 0.3540$	...	$\sigma = 7.01$ CR = 16.83% X = 73.88% $F_{\text{BC}} = 0.01425$	$\sigma = 7.13$ CR = 17.29% X = 73.73% $F_{\text{BC}} = 0.0101$

Table 1. Selected Pareto solutions (4 solutions out of 30)

Solutions for the 30 non-dominated individuals and their corresponding objective functions are obtained as a result of the optimization. In this paper, Table 1 only lists Nos. 1, 2, 29 and 30 out of the Pareto solutions, and the others are abbreviated because of spatial limitations. Meanwhile, in order to understand these solutions, they are arranged in increasing order of the standard deviation of average indoor humidity. As indicated, individuals with a combination value of 7 are the highest ranked Pareto

solutions since those individuals employ more moisture-conditioning material, thus enhancing the effectiveness of the moisture buffer. By contrast, most of the individuals with a combination value of 2 are at the lowest rank of Table 1 since only their ceilings featured moisture-conditioning material, thus lessening the effect of the moisture buffer. In comparison with the top ranked individuals, the lowest ones (such as Nos. 29 and 30) demonstrate higher standard deviation (worse humidity environment) but have lower values in terms of initial cost (more economic) and average relative indoor humidity (worse humidity environment).

## CONCLUSIONS

In order to create a reliable and safe environment in terms of indoor humidity, humidity control is considered crucial. Since adsorption materials are effective at performing humidity control, it is essential to develop suitable methods that can be employed at the conceptual design stage to assist designers in achieving better environmental design for indoor humidity both efficiently and quantitatively. This study proposed the multi-objective optimum design method to address the optimum utilization of moisture-conditioning material while both environmental and economic aspects are taken into account. Then, a theoretical case study was carried out in order to confirm the effectiveness of this optimum design model. Finally, the multi-objective genetic algorithm identified the Pareto solutions effectively, which are useful in both the decision-making stage and demonstrating the trade-off relationships between multi-objective functions. However, additional studies are necessary to focus on the application to the practical situations case by case while considering both the exterior and interior environment in the future.

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## NOMENCLATURE

$C_p$ : specific heat capacity at constant pressure [kJ/kg/K];  $\rho$ : density of air [kg/m<sup>3</sup>]; Q: airflow rate [m<sup>3</sup>/s]; k: number of heat source; n: number of moisture source; x: position within space; K: total number of heat sources; N: total number of moisture sources;  $\alpha'$ : mass convection coefficient [kg/m<sup>2</sup>/s];  $\alpha$ : convective heat transfer coefficient [W/m<sup>2</sup>/K]; L: latent heat of vapor [kJ/kg]; A: area of material arranged [m<sup>2</sup>]; m: mass of material [kg]; **Subscript** i: time step; D: air adjacent to material; S: material surface