EMBEDDED DETAILED ANALYSIS
INTEGRATING CFD WITH MULTIZONE METHODS OF BUILDING AIRFLOW ANALYSIS

James W. Axley†

School of Architecture, Yale University, P.O. Box 208242, New Haven, CT 06520-8242, USA

ABSTRACT
Methods of computational fluid dynamics (CFD) have been applied to predict the details of air, contaminant and thermal transport within isolated building zones, yet zone transport processes do not occur in isolation – they result from and interact with transport from the larger building system in which they are embedded. Consequently, there is a growing interest in combining CFD models of individual zones within multizone models of enclosing building systems to more faithfully model both the larger building interactions and the intrazonal details. In the rush to combine these modeling capabilities, however, few have paused to consider the problem of embedded detailed analysis fundamentally.

A fundamental approach based on mathematically coupling the CFD equations governing microscopic airflow in the embedded zone(s) to port plane formulations of the multi-zone equations governing macroscopic airflow in the enclosing building system will be presented. Specifically, finite element approximations of both the Stokes Flow and the Reynolds Average Navier Stokes (RANS) equations will be coupled to port plane formulations of the multizone equations and applied to a test case problem to illustrate the advantages of the approach and, yet, the limitations of embedded analysis in general.

KEYWORDS
Computational fluid dynamics (CFD), Multi-zone airflow analysis, Mechanical power balances

INTRODUCTION
In an attempt to predict the airflow details within individual zones while properly accounting for their interaction with the bulk airflows of the remaining building system a number of researchers have investigated embedding detailed CFD (microscopic) models of selected zones within multi-zone (macroscopic) models of whole building systems, Figure 1 (Gao 2002; Lorenzetti, Jayaraman et al. 2003; Mora, Gadgil et al. 2003; Chen and Wang 2004). To-date, however, embedded CFD analysis has been approached using ad hoc computational coupling strategies between largely independent multi-zone and CFD computer programs. Furthermore, these efforts have been directed to specific practical applications rather than to more fundamental studies directed to establish the inherent strengths and limitations of embedded detailed analysis and the coupling strategies used to affect the analysis. The research reported here presents a more fundamental approach to embedded detail analysis based on mathematically coupling the governing macroscopic equations to finite element approximations of the microscopic equations via direct assembly of the Jacobian matrices of each.

† Corresponding Author: +1 203 432 2283, Fax: +1 203 432 7175
E-mail address: james.axley@yale.edu
The coupled embedded detail model equations may then be solved using well-established numerical methods based on Newton’s Method (Kelley 2003).

![Diagram of a representative embedded detail model with figures illustrating the micro and macro variables at the coupled interface and two micro domain modeling strategies.](image)

Figure 1. Schematic of a representative embedded detail model with figures illustrating the micro and macro variables at the coupled interface and two micro domain modeling strategies.

The definition of the coupling relations between the macro and micro domains is, of course, key yet it has led to much confusion in the emerging field of embedded detail analysis principally because the system variables of the conventional approach to multizone analysis are defined in terms of pressures at discrete nodes within building zones rather than at zone boundaries. The port plane approach to macroscopic airflow analysis (Axley and Chung 2005; Axley and Chung 2005; Axley 2006), on the other hand, defines system variables in terms of spatially averaged port plane pressures \( \bar{p} \) and velocities \( \bar{v} \) at zone boundaries that may be related to the microscopic variables of pressure \( p \) and velocity \( v \) at the micro-macro coupled boundaries. This is not to say that the definition of these coupling relations is straightforward. Microscopic pressure and velocities will, in general, vary with the local dimensions \( (r,s) \) at these boundary while the port plane variables are, by definition, single-valued, Figure 1.

**METHODS**

To approach embedded detail analysis by coupling the governing macroscopic and microscopic equations mathematically and then solving the resulting system of coupled equations using well-establish solution methods for nonlinear systems of equations one needs appropriate computational tools. Fortuitously, the COMSOL simulation environment (COMSOL 2005) supports the coupling of algebraic equations to finite element approximations of partial differential equations, in general, and, more specifically, of the partial differential equations associated with Stokes Flow and the RANS using the standard \( k-c \) turbulence model. This simulation environment was, thus, used in this study to directly assemble the Jacobian matrices of a) the governing multizone equations for the macro domain, b) the finite element approximation of CFD equations for the micro domain, and c) the coupling relations between these domains and then solve these equations using variants of Newton’s Method.

To apply this analytic approach, one must clearly define the boundary of the microscopic domain and, thus, the boundary shared with the macroscopic domain and define the associated coupling relations.
**Definition of Micro Domain**

In this project two generic approaches to the definition of the microscopic flow domain have been considered – an inclusive model that includes the inflow and outflow paths to a zone within the microscopic domain and an exclusive model limited to the selected zone only, Figure 1. (Our limited experience to-date indicates that RANS $k$ - $\varepsilon$ embedded detail models are more likely to converge if inflow and outflow paths are included within the microscopic domain while Stokes Flow embedded models proved to be insensitive to the inclusion of these flow paths.)

**Micro/Macro Coupling Relations**

Distinguishing micro-to-macro couplings from macro-to-micro, one may first impose the straightforward relations for the former that, in fact, define the port plane variables of pressure $\hat{p}_i$ and velocity $\hat{v}_i$:

Micro-to-macro

$$\hat{p}_i = \frac{1}{A_i} \int p dA \quad \text{and} \quad \hat{v}_i = \frac{1}{A_i} \int v dA$$

(1, 2)

where $v$ is normal to the port plane $i$ and the cross-sectional area of the port plane is $A_i$.

The macro-to-micro coupling definitions are, on the other hand, indeterminate as a) pressure and velocity will vary across the area of the micro-macro boundary in some generally unknown manner and b) velocity components tangent to the boundary, $u$ and $w$, and turbulence parameters, $k$ and $\varepsilon$, remain undefined in macroscopic theory. Here, however, for inclusive RANS $k$ - $\varepsilon$ models pressure at the boundary was assumed to be uniform:

Macro-to-micro pressure

$$p(r, s) \approx \hat{p}_i$$

(3)

While for exclusive embedded Stokes Flow studies model studies, port plane velocity distributions were also modeled as uniform:

Macro-to-micro normal velocity

$$v(r, s) \approx \hat{v}_i$$

(4)

In a number of studies velocity distributions normal to the micro domain boundary were also assumed to vary quadratically to approximate developed laminar flow conditions or to vary by power law relations with exponents of $1/7$ to approximate turbulent flow conditions.

While the coupling strategy used for inclusive RANS $k$ - $\varepsilon$ models avoided the need to define a normal velocity coupling relation, the indeterminacy of coupling the tangential velocity components proved to have a significant impact on the fidelity of embedded analysis.

**RESULTS OF MODELING STUDIES**

To investigate the proposed approach, a hypothetical symmetric test case building was devised that represents a generally typical flow topology of competing parallel flow paths (i.e., that are invariably found in building airflow systems). This test case building also provides partial self-evident validation of computed results. The basic geometry of this hypothetical building and the associated macroscopic variables used are presented in Figure 2 and Table 1. Spatially averaged port plane pressures $\hat{p}$ and velocities $\hat{v}$ are represented with a double character notation “pp” (i.e., pp1 to pp12) and “vp” (i.e., vpa to vpf). A 3D representation of the test building is illustrated in the upper corner of Figure 2 – it is a simple 3 m extrusion of the building plan with all openings centered at a height of 1.5 m.
Given the symmetric geometry and boundary conditions assumed (i.e., \( pp8 = pp10 \)), one must expect that the bulk airflow rates through the paired zones will be identical and airflow from the lower zone 3 to the upper zone 2 will be negligibly small (i.e., \( vpf = 0 \)). Thus, deviation from these symmetric flows will directly reveal the failure of any analytical strategy to model airflow faithfully, independent of magnitudes. Both 2D and 3D analyses were investigated, but here the discussion will be limited to the 3D results.

**Full Macroscopic \\& Microscopic Modeling Studies**

To provide analytical benchmarks, the full test case building was modeled both macroscopically and microscopically for isothermal conditions and a 1 Pa overall driving pressure difference. Two macroscopic models were considered – both based on port plane approaches. The first followed conventional practice in that a hydrostatic field assumption was used to account for intra-zone pressure distributions – here, pressure equality between all zone ports. The second modeled flow in each of the three zones using mechanical power balances augmented by a single additional field assumption for each zone – i.e., here, pressure equality between a single pair of port plane pressures.

Dissipation in each of the restricted flow paths was modeled using friction loss factors based on Idelchik’s correlations for thick-edged orifices under turbulent flow conditions (Idelchik 1994), Table 1.

| Flow Path | Thickness (m) | Area \( A_j \) (m²) | Friction Loss Factor
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 1</td>
</tr>
<tr>
<td>a</td>
<td>0.15</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>b</td>
<td>0.15</td>
<td>0.75</td>
<td>0.0225</td>
</tr>
<tr>
<td>c</td>
<td>0.15</td>
<td>0.75</td>
<td>0.0225</td>
</tr>
<tr>
<td>d</td>
<td>0.15</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>e</td>
<td>0.15</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>f</td>
<td>0.15</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Using these friction loss factors, port plane pressures and velocities were computed for both conventional and power balance full multi-zone models for the Case 1 geometry having relatively large openings \( Ab \) and \( Ac \) and for the Case 2 geometry having relatively small openings. Figure 2 presents the results for the conventional multi-zone approach for Case 1. Figure 3 presents the detailed results on a horizontal mid-height plane for full microscopic analysis using finite element solutions of the RANS \( k - \varepsilon \) equations. The Case 2 results will not be considered here due to publication limits.

As expected, both macro and micro analyses produce symmetric flow results with negligibly small cross-flows through flow path f. Furthermore, the micro results show that as the flow splits to pass through openings b and c it develops outward trajectories that result in complementary asymmetric flow structures within zones 2 and 3, respectively, with large recirculation regimes adjacent to the wall separating these zones – again, as expected. If one examines the pressure distributions revealed by the grey scale, one can immediately see that pressures within each of the zones are nearly uniform (i.e., or here, for the isothermal conditions, hydrostatic) as is assumed in conventional multi-zone analysis.

Finally, Figure 4 presents a side-by-side comparison of port plane velocity (top) and pressure (bottom) results from macro conventional analysis, macro power balance analysis and micro RANS \( k - \varepsilon \).
analysis. While the port plane velocities are comparable, those computed from the RANS $k$-$\varepsilon$ results are consistently greater than the macro results. The port plane pressures are presented in the order that follows the upper flow path with dashed and dotted lines illustrating the predicted conventional and power balance profiles, respectively. The former shows uniform pressures within zones 2 and 3 (i.e., due to the hydrostatic assumption) while the power balance predicts pressure increases in these zone – apparently exaggerating the static regain associated with inflow jets. The RANS $k$-$\varepsilon$ shows no evidence of this.

Unfortunately, each of these three analytical methods is compromised by simplifying assumptions, thus no one of them may be assumed to provide unassailable results.

Figure 2. Case1: Conventional Multi-Zone Analysis Results: Spatial averaged port plane pressures (pp in Pa) and velocities (vp in m/s) computed for Case 1 geometry.

Figure 3. Full 3D RANS $k$-$\varepsilon$ Analysis: Detailed airflow velocities (arrows) and pressure variation (grey scale) for a 3D RANS $k$-$\varepsilon$ finite element analysis of the test case building - computed in 4.4 h
using a finite element mesh of 43,266 elements on a fast 64 bit PC. Spatial averaged port plane pressures (pp in Pa) and velocities (vp in m/s) computed from the microscopic results are also shown.

Figure 4 Side-by-side comparisons of port plane results computed from the RANS $k$-$\varepsilon$, multizone conventional, and multizone power balance analyses.

Embedded Modeling Studies

Following the procedures outlined above, embedded CFD models were formulated and analyzed a) for the Case 1 geometry, b) modeling zone 3 with finite element approximations of the RANS $k$-$\varepsilon$ equations, and c) modeling zone 1, zone 2, and flow paths b and d with each of two multizone models. Figure 5 presents the results of these studies. As the governing macro, micro and coupling equations were fully coupled and solved using well-established solution procedures and the mesh of the micro domain was refined to achieve nearly grid-independent results, the computed approximations must be expected to represent solutions to the posed embedded CFD problems (i.e., within a specified solution accuracy).

Examining these results, neither the predicted zone 3 micro flow results nor the macroscopic bulk flow results are consistent with reasonable expectations or similar to that predicted by the full 3D RANS $k$-$\varepsilon$ or multizone analyses, Figures 2 and 3. That is to say, the embedded detail models do not faithfully model flow detail in the microscopic zone nor do they model the larger macroscopic flow structure correctly. Examining port plane velocity magnitudes reveals, in addition, continuity of flow is violated with the inflow volumetric flow rate exceeding outflows by about 4%. Furthermore, even this simple embedded detail analysis proved to be computational demanding. These results were computed in 10 hours and 11 hours, respectively, using finite element meshes of 25,414 elements on a fast 64 bit PC with 8 GB of memory.

Given the poor performance and computational demands of the embedded RANS $k$-$\varepsilon$ models, additional studies were completed using the Stokes Flow equations (i.e., the Navier Stokes equations assuming the density is negligibly small) for zone 3 and, again, conventional and power balance multizone models for the enclosing zones and openings. Figure 6 presents the results for the latter. While the predicted micro domain flow structure lacks the asymmetry and recirculation, the dominant stream tube velocities are similar in magnitude to those predicted in the full RANS model and the macro
results are symmetric and satisfy continuity as expected. Furthermore, these grid-independent results were computed in less than 4 minute using a coarse mesh of 2,463 finite elements on a fast 64 bit PC.

As embedded Stokes Flow models may be solved with little computational effort, because the Stokes Flow equations are linear, they may provide the means to rapidly compute a crude estimate of room airflow structure and, perhaps, contaminant dispersal delays associated with these airflows. Furthermore, as an embedded exclusive Stokes Flow model effectively presents minimal resistance to airflow, it appears to be compatible with multizone models that neglect zone resistance and thereby does not compromise the macro flow results of the embedded detail model analysis.

Figure 5 Results of an embedded 3D RANS $k$-$\varepsilon$ model with port plane formulations of enclosing conventional multizone model (top) and power balance multizone model (bottom).
CONCLUSION

While CFD and multizone methods appear to have complementary capabilities, there is no compelling reason to believe that, when combined for embedded detail analysis, improved or even correct results will be obtained. Here a formulation is presented using unambiguous mathematical definitions of the coupling relations between the embedded CFD and multizone models made possible by using a port-plane approach to the multizone model. Though the micro-to-macro coupling is straightforward, macro-to-micro coupling must remain indeterminate given multi-zone models do not account for non-normal airflows to port planes or for turbulence variables used by some embedded CFD models.

A hypothetical test problem configured to be sensitive to the inability of multizone models to account for non-normal inflow velocity components and a less obvious shortcoming of CFD models – their tendency to model flow resistance differently and thus incompatibly with multi-zone models – indicates that embedded detail analysis can not, in general, be expected to faithfully model flow detail in the CFD embedded zone nor model the larger macroscopic bulk flow structure correctly. As these results place the value of embedded detail analysis into question, additional research is clearly needed to identify the special cases when embedded detailed analysis may be expected to be reliable.

ACKNOWLEDGEMENTS

The work reported here was completed with support from the U.S. DOC National Institute of Standards and Technology (NIST) with the assistance of research assistant Daniel Chung.

REFERENCES