Effects of Geometrical Shape of Roofs on Natural Convection for Winter Conditions

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SUMMARY

In the practical applications, roofs of buildings can be in different shapes depending on architectural design of building or climate. Some of these building roofs can be classified as gambrel, saltbox and gable roofs. In the present study, we investigated the natural convection heat transfer and fluid flow inside the gambrel, gable and saltbox roofs for winter boundary conditions. With this aim, the identified roofs are compared with each other from the heat transfer and flow field point of view. Effects of Rayleigh number also tested in each type. Results are presented with streamlines, isotherms, local and mean Nusselt numbers. It is aimed that this study will help for designer to show more efficient ways of saving energy.

Keywords: natural convection, roof, winter conditions

INTRODUCTION

Roofs protect the building from environmental effects such as rain, snow and wind or particle matter in air. It also prevents the heat losses from environment to the room and vise versa. One can see different types of roofs as gambrel, gable, saltbox, and shed roofs etc. in urban or rural areas. These types of the roofs are chosen according to decorative view or type of the building. Both winter and summer day conditions natural convection heat transfer occurs inside the roof due to buoyancy forces and temperature difference between ceiling of the room and sloping wall of the roof.

Investigation of the natural convection heat transfer and fluid flow inside the roof is important to save energy and reduce to heating or cooling load of the building. Thus, cost of energy of the building will be decrease. Natural convection in roofs is analyzed mainly for gable roofs in the literature. Asan and Namli [1,2] investigated the natural convection heat transfer in gable roofs for both summer and winter day boundary conditions using finite difference techniques. They indicated that both aspect ratios of the roof and temperature boundary conditions are important parameters on temperature and flow fields inside the roof. Varol et al. [3] made a numerical solution for gambrel roofs for both summer and winter conditions. The same method applied the solution of natural convection inside the Saltbox roofs [4] and shed roofs with or without eave in summer conditions [5]. Natural convection in trapezoidal enclosure with offset baffles investigated by Moukalled and Acharya [6]. They tested the effects of baffle on buoyancy-induced flow and thermal fields in the trapezoidal shaped attics. They observed that conjugate baffles make important effect on natural convection and can be used control parameter for heat transfer.
The main purpose of this study is to examine the natural convection heat transfer in the different types of the roof. These roofs are chosen as gambrel, gable, saltbox and compared with each other from the heat transfer point of view. The bottom area of the roofs is equal in each type to compare occurred heat transfer in the roof.

**DEFINITION OF TREATED ROOFS MODELS**

Three different roofs models were chosen to compare the effects of roof shape on temperature and flow field of natural convection. Figure 1 shows physical models for treated roof types. Inclination angle is the most important parameter for a roof which is chosen based on meteorological data of the region. To compare roof types in this study, inclination angle of roofs are chosen according to climate of the East Anatolian region of Turkey. Thus, main inclination angles are chosen 52° and 18° for gambrel, gable and saltbox roofs, respectively. Comparisons are performed depends on winter conditions. Thus bottom wall (ceiling of the room) behaves as heater and temperature of the inclined wall is colder than that of bottom wall.
Left part of the roof according to symmetry line was chosen as computational domain in each type except saltbox roof. Two-dimensional, steady-state and laminar solution were accepted. Further assumption was made that flow is incompressible and Newtonian. Thus, governing equations can be written as follows:
\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \Psi = \frac{\psi}{L}, \quad \Omega = \frac{\omega(L)^2}{\nu}, \quad \theta = \frac{T - T_c}{T_h - T_c} \] (1)

\[ u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}, \quad \omega = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right), \quad Ra = \frac{\beta g (T_h - T_c) L^3 \Pr}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}. \] (2)

Based on the dimensionless variables above governing equations (stream function, vorticity and energy equations) can be written as

\[ -\Omega = \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} \] (3)

\[ \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} = \frac{1}{\Pr} \left( \frac{\partial \Psi}{\partial Y} \frac{\partial \Omega}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} \right) - Ra \left( \frac{\partial \theta}{\partial X} \right) \] (4)

\[ \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y} \] (5)

The numerical method used in the present study is based on finite difference method to discretize the governing equations (Eqs. 3-5) and the set of algebraic equations are solved using Successive Under Relaxation (SUR) technique. The solution technique is well described in the literature [7,8] and has been widely used to solve natural convection equations. The convergence criterion $10^{-4}$ is chosen for all depended variables and 0.1 is taken for under-relaxation parameter. The present solutions are compared with the known results for triangular shaped roofs from the open literature. As indicated in the Figure 2, it was observed that these results of the present code are in very good agreement with the literature [2,9,10].

![Figure 2. Comparison of obtained results with literature.](image)

As indicated above, the problem is defined for winter boundary conditions. The physical boundary conditions are illustrated in the physical model (Figure 1) and they can be defined as follows:
On the inclined surfaces, \( u=v=0, T_U=T_{\text{cold}} \) \hspace{1cm} (6)

On the bottom wall, \( u=v=0, T_B=T_{\text{hot}} \) \hspace{1cm} (7)

On the symmetry line, \( u=0 \), vertical wall, \( u=0 \), \( \partial T/\partial x = 0, \partial T/\partial x = 0 \) \hspace{1cm} (8)

Calculation of the local Nusselt number was performed by

\[
Nu_x = -\left. \frac{\partial \theta}{\partial y} \right|_{y=0}
\] \hspace{1cm} (9)

**RESULTS AND DISCUSSION**

Numerical analyses of natural convection temperature and flow fields for different types of the roofs have been performed in this study for different Rayleigh numbers. Streamlines and isotherms are shown in Figure 3 for different type of roofs in winter conditions at \( Ra=10^6 \). Thus, the bottom area of the roofs is chosen as equal. As can be seen from the figure, double circulation cells were formed in type of saltbox roof. However, in other types of the roofs single cell was obtained.

![Streamlines and isotherms for different roof types](image)

*Figure 3. Flow field by streamline (On the left) and temperature distribution by isotherms for winter day conditions at \( Ra=10^6 \) a) Saltbox, b) Gambrel, c) Gable*
Figure 4. Variation of mean Nusselt number with Rayleigh number for different types of the roof

Figure 4 shows the mean Nusselt numbers which defines the overall heat transfer. They have given for different types and Rayleigh number. As can be seen from the figure, heat transfer mode is mainly conductive at low Rayleigh numbers. Thus, mean Nusselt number becomes constant with increasing of Rayleigh number up to $10^5$. For higher Ra numbers, mean Nusselt number increases with increasing of Ra number due to domination of convection mode of heat transfer. The figure shows that, the highest mean Nusselt number is obtained when saltbox roof is chosen due to strong flow strength. It is interesting result that when gambrel roof was chosen same values of mean Nusselt number was obtained between saltbox and gambrel roof at the highest Rayleigh number. Figure 5 presents the local Nusselt number over heated wall of roofs. Thus, variation of local Nusselt number can be comparable for different types of the roofs. Figure presents that, local Nusselt number shows almost sinusoidal shaped variation for gable roofs and its value smaller than that of saltbox roof. Again, gambrel roof shows wavy variation but its value is smaller at the left corner due to long distance between hot and cold walls. Variation shows monotonically increasing for saltbox roof and highest local Nusselt numbers are obtained for this type of roof.
CONCLUSION

Temperature distribution and flow fields of natural convection are presented for three types of the roofs using a numerical technique. It was observed that single cell was formed for gable and gambrel roofs but double circulation cell was formed for saltbox roof. It means that the flow strength is higher for these types of the roof. The lowest heat transfer was performed for gambrel roof at the low Rayleigh numbers. On the contrary, almost same values are formed for both saltbox and gambrel roofs at the highest Rayleigh number.

NOMENCLATURE

g gravitational acceleration (ms\(^{-2}\))
Gr Grashof number
H height of roof (m)
L length of bottom wall (m)
Nu Nusselt number
Pr Prandtl number
Ra Rayleigh number
T temperature (K)
u, v axial and radial velocities (ms\(^{-1}\))
x, y cartesian coordinates (m)
X, Y non-dimensional coordinates

Greek Letters

\(\nu\) kinematic viscosity (m\(^2\)s\(^{-1}\))
\(\Omega\) non-dimensional vorticity
\(\theta\) non-dimensional temperature
\(\beta\) thermal expansion coefficient (K\(^{-1}\))
\(\alpha\) thermal diffusivity (m\(^2\)s\(^{-1}\))
\(\Psi\) non-dimensional streamfunction

REFERENCES