Control of temperatures in air-conditioned indoor spaces using reduced order model

A. Sempey, C. Inard, C. Ghiaus, C. Allery
LEPTAB, Pôle sciences et technologie, France

ABSTRACT

Real-time control of comfort in indoor spaces needs models of temperature distribution and air-velocity velocity field. Currently, a one zone model is used assuming that the temperature is homogenous in the whole space. If the heterogeneity of air velocity field and the temperatures distribution is to be taken into account by the control system, a direct or indirect measurement of the temperature in the occupancy zone is needed. The purpose of this study is to control the temperature in the occupancy zone by integrating a low order model into a feedback control loop. The state space form of the reduced order model allows us to estimate the temperature in the occupancy zone, without a direct measurement, and to feedback it to the controller. The state estimator formed gives the possibility to take into account the difference of temperature between the sensor zone and the occupancy zone in order to control thermal conditions.

1. INTRODUCTION

In air-conditioned spaces, the air velocity field and the temperature distribution are not uniform with significant implications on the thermal comfort. Nowadays, this characteristic is generally not taken into account by the control system which also has consequences on energy consumption. For real-time control, rooms are equipped with only one sensor which is placed outside the occupancy zone because of the use of this zone. In consequence, the temperature estimation in the occupancy zone is necessary. Sempey et al. (2007) obtained a reduced order model by using a fixed flow field and the proper orthogonal decomposition. A reduced order model was built for each airflow configuration, which makes necessary the interpolation in order to smooth the transition between different models. The models for each airflow configuration are linear and may be written in state-space form, which is suitable for the modern control theory. These models are used to estimate the temperature into the occupancy zone by taking into account only one sensor and to design an internal model controller.

2. THEORY

1.1 The state estimator

For each configuration of the flow field, Sempey et al. (2007) built a reduced order model of the form:

\[
\begin{align*}
\dot{a} &= A_r a + B_r u \\
\dot{\theta} &= C_r a + D_r u
\end{align*}
\]

where

- \(a\): temporal coefficient,
- \(A_r\): reduced order state matrix,
- \(B_r\) and \(D_r\): reduced order inlet matrix,
- \(C_r\): reduced output matrix,
- \(u\): input vector.

The low order model (1) gives the temperature of any point of the room. In particular, the temperature in the occupancy zone, \(\theta_{mes}\), can be computed by:

\[
\begin{align*}
\dot{a} &= A_r a + B_r u \\
\dot{\theta}_{mes} &= C_r a + D_r u \\
\end{align*}
\]

The error made by the reduced order model (2) can be evaluated by comparison with the measured temperature. The modern control theory proposes to correct the state space system by adding a term proportional to the error evaluated for the measured point. The state estimator is then given by:

\[
\begin{align*}
\hat{a} &= \hat{A}_r a + \hat{B}_r u + L (\hat{\theta}_{mes} - \hat{\theta}_{mes}) \\
\hat{\theta}_{mes} &= \hat{C}_r \hat{a} + \hat{D}_r \hat{\theta}_{mes} \\
\end{align*}
\]

where \(^\wedge\) stands for estimated quantities.

The estimator matrix \(L\) aims at decreasing the difference between the reduced order model (2) and the state estimator (3), which amounts to correcting temporal coefficients. By introducing \(a_e = a - \hat{a}\) and \(\theta_e = \theta_{mes} - \hat{\theta}_{mes}\), we finally have to solve a minimization problem with the constraint:

\[
\begin{align*}
\hat{a}_e &= \hat{A}_r a_e + \hat{L} C_r \hat{a}_e \\
\hat{\theta}_e &= \hat{C}_r \hat{a}_e \\
\end{align*}
\]
and the cost functional:
\[ J = \int \left[ \dot{a}^T \Lambda a + \int_{r_{\text{ref}}}^t e^T \Lambda e dt \right] \]

where \( \dot{a}^T \) is the transpose vector of \( a \).

Optimal control theory provides a way to select the “best” matrix \( \Lambda \) in order to minimize the cost-functional (5). The expression of \( \Lambda \) (Bewley and Liu (1998)) is then given by:
\[ \Lambda = -Y C^T \]

where \( Y \) is the solution of the Riccati equation:
\[ A_r Y + Y A_r^T - Y C^T C Y + 1 = 0 \]  

(7)

The estimator is then able to predict in real-time the temperature in any point of the mesh by:
\[ \hat{\theta} = \sum_{i=1}^{n} w_i \hat{\theta}_i \]

where:
- \( \hat{\theta} \): estimated temperature,
- \( w_i \): weight coefficient,
- \( \hat{\theta}_i \): estimated temperature for the configuration \( i \),
- \( n \): number of configuration.

1.2 Design of the controller

The internal model control is used to design a controller. This method relies on the internal model principle, which states that control can be achieved only if the control system encapsulates some representation of the process to be controlled. In particular, if the control scheme has been developed based on an exact model of the process, then perfect control is theoretically possible (Tham (2002)). The perfect controller \( G_e \) is given by:
\[ G_e = \tilde{G}_p^{-1} \]

(10)

where \( \tilde{G}_p^{-1} \) is the inverse of the process-model.

In practice, the process model is not perfect and is not totally invertible. However, this philosophy leads to the strategy of the internal model control, which the structure is depicted in Figure 1.

![Figure 1: Scheme of the internal model control](image)

To improve robustness, the effects of process model mismatch should be minimized. Since the discrepancies between process and model behaviour usually occur at the high frequency end of the system frequency response, a low-pass filter \( G_f \) is usually added. The internal model controller (IMC) becomes:
\[ G_{\text{IMC}}(s) = G_c(s)G_f(s) \]

(11)

The block diagram in Figure 1 can be reduced to a conventional closed loop structure in order to build a conventional controller:
\[ G_c(s) = \frac{G_{\text{IMC}}(s)}{1 - \tilde{G}_p(s)G_{\text{IMC}}(s)} \]

(12)

The process, expressed as state space system of reduced order (Sempey et al. (2007)), can be transformed into:
\[ \tilde{G}_p = \frac{N(s)}{D(s)} \]

(13)

It is necessary to transform the transfer function \( \tilde{G}_p \) into:
\[ \tilde{G}_p = \frac{N_-(s)N_+(s)}{D(s)} \]

(14)

where:
- \( N_-(s) \): invertible part of \( \tilde{G}_p \),
- \( N_+(s) \): non invertible part of \( \tilde{G}_p \).

In this case the IMC controller can be written as (Brosilow and Joseph (2002)):
\[ \tilde{G}_{\text{IMC}} = \frac{D(s)}{N_-(s)N_+(s)(-T_f s + 1)^r} \]

(15)

where:
- \( T_f \): time constant of the low pass filter,
- \( r \): relative order of \( N(s) \) and \( D(s) \).

A controller is designed for each configuration which requires to change the controller with the airflow configuration. Consequently, a strategy to adapt controllers...
to all configurations is applied exactly in the same way as for the state estimator:

$$U(s) = \sum_{i=1}^{n} w_i U_i(s)$$

(16)

where:

- $U$: command,
- $w_i$: weight coefficient,
- $U_i$: command computed with the controller of the configuration $i$,
- $n$: number of configuration.

Last step consists in integrating the controller in a closed loop structure including the state estimator to control the temperature in the occupancy zone (Figure 2).

3. RESULTS AND DISCUSSION

3.1 Estimation of the temperature in the occupancy zone

In order to evaluate the accuracy of the models, the root mean square error between the reference results obtained with CFD and the results obtained by using the reduced model is computed:

$$\text{Rmse} = \sqrt{\frac{1}{n_x} \sum_{x} (x_{\text{ref}} - x)^2}$$

(17)

where:

- $x_{\text{ref}}$: reference, i.e. CFD results;
- $n_x$: number of cells for steady simulations, or number of time steps for transient simulations.

In order to test the state estimator, the results of the full CFD model are considered as measured values for the measured temperature $\theta^{\text{mes}}$. Figure 4 gives the results of the state estimator for an outlet temperature of 21 °C and a temperature step of 1 °C. Figure 4d shows that the transient error at the point $P_4$ is zero which emphasizes the efficiency of the state estimator at the measured point. Results are also very good for the point $P_3$ (Fig 4c), where the $\text{Rmse}$ is equal to 0.090 °C. However, $\text{Rmse}$ increases for the points $P_1$ and $P_2$ at 0.158 °C and 0.324 °C, respectively. Figure 4 highlights that the error between the reduced order model and the full CFD model for $P_1$ has an opposite sign than the error for $P_4$. That is why the state estimator decreases the accuracy.
for points $P_1$ and $P_2$. Nevertheless, values of $R_mse$ remain acceptable for $P_1$, and are explicable for $P_2$ by the high temperature gradient in the jet. In steady state, the $R_mse$ are slightly lower. This remark has to be associated with the method to design the state estimator which minimizes a cost function on the transient period but not on the steady state.

To conclude, the state estimator is able to predict the temperature in the occupancy zone with a sufficient accuracy. Consequently, it can be integrated as an indirect measure of the temperature in the occupancy zone in addition to inlet temperature that is usually used.

### 3.2 Control of the temperature in the occupancy zone

This part deals with the comparison of two controllers for the control of the temperature at the point $P_2$.

The first one is a PI controller build with the IMC method. For each airflow pattern, the first step consists in identifying a first order model. So the reduced order model is truncated with the Hankel norm minimization method in the same way as Palomo Del Barrio et al. (2000). A unique PI controller is then built in keeping the minimum gain and the major integral time.

A second controller of order 7 is directly designed by using the IMC method with the seventh order model of the room for each airflow pattern. Contrary to the PI controller, this one is adapted for each case with the weighted law described in the section (1.2).

The results of both the controllers are given in Figure 5: Control of the occupancy zone temperature, $(-)$ set point temperature, $(-+)$ occupancy zone temperature, $(-)$ outlet temperature, with (a) the PI controller, (b) the seventh order controller.

The results of both the controllers are given in Figure 5: Control of the occupancy zone temperature, $(-)$ set point temperature, $(-+)$ occupancy zone temperature, $(-)$ outlet temperature, with (a) the PI controller, (b) the seventh order controller.
5 for five different steps of set point temperature. Even if the differences between both controllers are not very clear, several facts are to be pointed out. At first the worse result of the seventh order controller for the step beginning at time 6500 s is due to a non physical behaviour of the room model for the case b. Indeed, whereas an increase of the temperature is waited, this one starts with falling rapidly. In this case, the major lesson to learn is that the first order controller is more robust. However for all other cases, the high order controller shows a shorter response time and a better stability of the inlet temperature.

4. CONCLUSIONS

In this paper, a reduced order model has been used to design a state estimator able to predict the temperature in the occupancy zone in real time. Furthermore, the low order of the room model allows the use of modern control. A first attempt has been presented here by using the internal model strategy. Results are encouraging but have to be improved, in particular for the robustness of the model. In this way, the state space form of the reduced order model should turn out to be interesting to build optimal and robust controller. Last, this method gives the possibility to take into account the temperature stratification in room control which is one the main characteristic of large enclosure.

REFERENCES